Joint Image Formation and Anisotropy Characterization in Wide-Angle SAR

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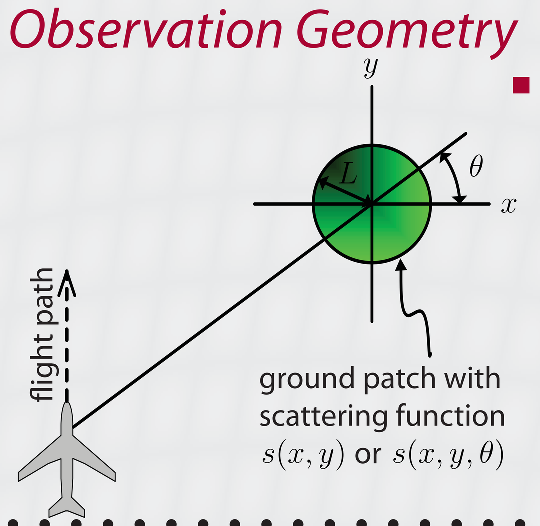
2-D Spotlight-Mode SAR

Observation Geometry



- A radar set mounted to the aircraft transmits pulses of electromagnetic energy containing many frequencies f with speed of propagation c towards the ground patch, and receives the scattered return from N angles along the flight path of the aircraft. The scattered returns are scaled in amplitude and shifted in phase by the complex-valued scattering function of the ground patch.

Phase History Measurements & Point Scattering

- synthetic aperture radar (SAR) measurements take the form:

$$r(f,\theta) = \iint_{x^2+y^2\le L^2} s(x,y)\exp\left\{-j\frac{4\pi f}{c}(x\cos\theta + y\sin\theta)\right\}dx\,dy$$

- but, "the manner of propagation at a given point is determined solely by the properties of the medium and the structure of the field in an arbitrarily small neighborhood of the point" (Keller, 1962); thus, with P discrete point scattering centers, the phase history measurement model is:

$$r(f,\theta) = \sum_{p=1}^{P} s(x_p,y_p)\exp\left\{-j\frac{4\pi f}{c}(x_p\cos\theta + y_p\sin\theta)\right\}$$

Anisotropy

- in principle, wide-angle apertures permit the reconstruction of images with high cross-range resolution but dependence of scattering on aspect angle, *anisotropy*, becomes a significant issue, resulting in the observation model:

$$r(f,\theta) = \sum_{p=1}^{P} s(x_p,y_p,\theta)\exp\left\{-j\frac{4\pi f}{c}(x_p\cos\theta + y_p\sin\theta)\right\}$$

- characterized anisotropy may be used as a feature for automatic target recognition and for improved image formation

Sparse Signal Representation

Overcomplete Expansion of Signals

- with an N-dimensional signal $\mathbf{r}$, and an overcomplete basis $\mathbf{\Phi} = [\phi_1 \;\; \phi_2 \;\; \cdots \;\; \phi_M]$, $M > N$, we would like to find a vector of coefficients $\mathbf{a}$ such that $\mathbf{r} = \mathbf{\Phi a}$
- there is an infinite subspace of solutions; we favor sparse solutions
- with additive noise: $\mathbf{r} = \mathbf{\Phi a} + \mathbf{n}$

The Sparse Signal Representation Problem

- the ℓ_k-norm of $\mathbf{a}$ is defined as $\|\mathbf{a}\|_k = \left(\sum_{m=1}^{M} |(a)_m|^k\right)^{\frac{1}{k}}$
- the ℓ_0-norm counts the number of non-zero entries in $\mathbf{a}$
- then, the sparse signal representation problem is:

noiseless: $\min \|\mathbf{a}\|_0$ s.t. $\mathbf{r} = \mathbf{\Phi a}$ additive noise: $\min \|\mathbf{a}\|_0$ s.t. $\|\mathbf{r} - \mathbf{\Phi a}\|_2 \le \delta$

- this is a combinatorial optimization problem

Greedy Methods and Relaxations

- matching pursuit: select best available basis vector greedily on every iteration
- basis pursuit: ℓ_1 relaxation solved by linear programming: $\min \|\mathbf{a}\|_1$ s.t. $\mathbf{r} = \mathbf{\Phi a}$
- SPARSIFYING REGULARIZATION: $\min_{\mathbf{a}} J(\mathbf{a}) = \|\mathbf{r} - \mathbf{\Phi a}\|_2^2 + \alpha\|\mathbf{a}\|_k^k$, $k < 1$ where α is a regularization parameter that trades off data fidelity and sparsity

Overcomplete Basis Formulation

Objective

- in joint image formation and anisotropy characterization, our goal is determining $s(x,y,\theta)$ from the phase history measurements:

$$r(f,\theta) = \sum_{p=1}^{P} s(x_p,y_p,\theta)\exp\left\{-j\frac{4\pi f}{c}(x_p\cos\theta + y_p\sin\theta)\right\}$$

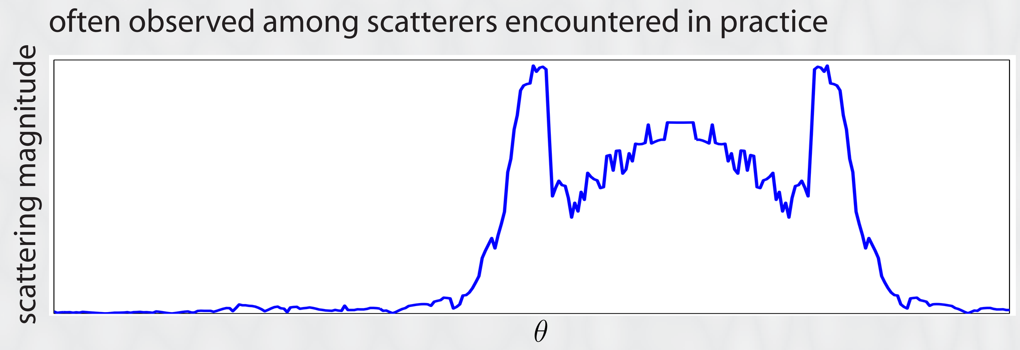
Overcomplete Formulation

- the proposed approach is to expand the scattering function for each scattering center as the sum of an overcomplete set of basis vectors

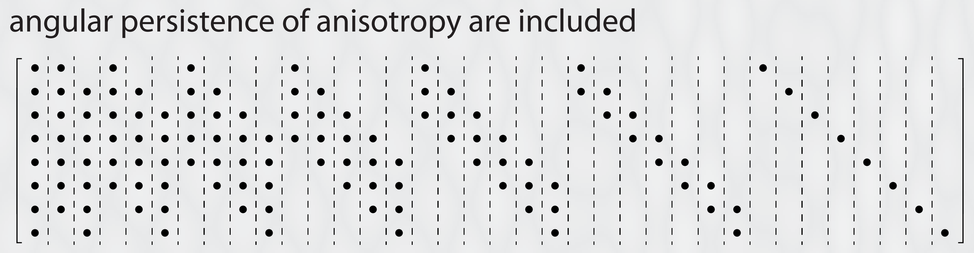
$$r(f,\theta) = \sum_{p=1}^{P}\sum_{m=1}^{M} a_{p,m}b_m(\theta)\exp\left\{-j\frac{4\pi f}{c}(x_p\cos\theta + y_p\sin\theta)\right\}$$

Choice of Basis Vectors

- contiguous intervals in aspect angle of non-zero scattering behavior are often observed among scatterers encountered in practice



- we choose the set of basis vectors such that all *widths* and *shifts* in the angular persistence of anisotropy are included

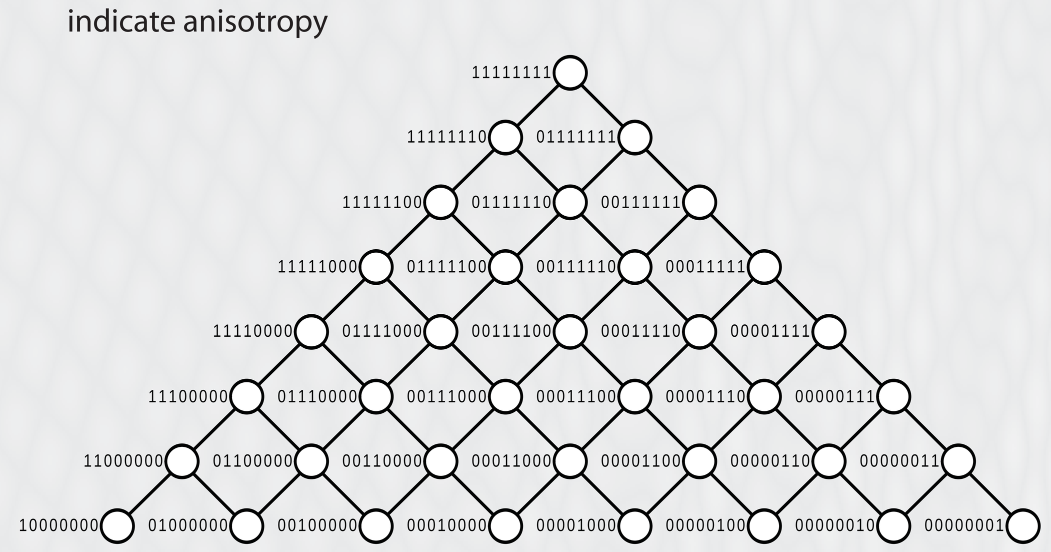


- the above diagram illustrates the overcomplete basis for $N = 8$ — dots indicate non-zero entries and spaces represent zero-valued entries; any pulse shape may be used for the basis vectors, e.g. rectangular, Hamming window, triangle, raised triangle, windowed Gaussian
- for this choice of overcomplete basis, the number of basis vectors M and the number of angle samples in the measurements N are related by:

$$M = \binom{N+1}{2} = \frac{1}{2}N^2 + \frac{1}{2}N$$

Graph-Structured Interpretation

- the overcomplete basis has an intuitive graph-structured interpretation, given the name *basis graph*, illustrated below for $N = 8$ and rectangular pulse shape, where nodes represent basis vectors and labels to the left indicate anisotropy



- the basis vector at the root is isotropic; top-to-bottom corresponds to coarse-to-fine angular persistence and left-to-right corresponds to shifts of the center angle of anisotropy
- there are P coexisting basis graphs, one per spatial location

Sparsifying Regularization

Regularization Cost Function Approach

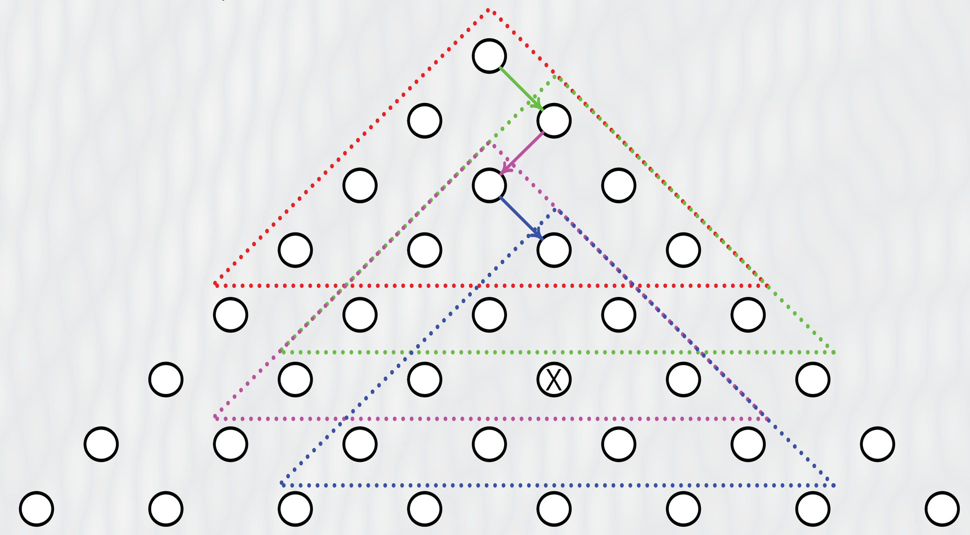
- by using the overcomplete basis $\mathbf{\Phi}$ and the regularization cost function $J(\mathbf{a}) = \|\mathbf{r} - \mathbf{\Phi a}\|_2^2 + \alpha\|\mathbf{a}\|_k^k$, $k < 1$, we treat all spatial locations jointly within one system of equations — taking interactions among scatterers into account
- we use data from the full aperture — we do not break things up into subapertures
- the approach is more flexible than parametric methods, but still incorporates prior information about anisotropy through the choice of basis vectors

Quasi-Newton Method

- the cost function is made differentiable at 0 through the approximation $J_\epsilon(\mathbf{a}) = \|\mathbf{r} - \mathbf{\Phi a}\|_2^2 + \alpha\sum_{i=1}^{M}\left((a)_i^2 + \epsilon\right)^{k/2}$
- with $\mathbf{H}(\mathbf{a}) = 2\mathbf{\Phi}^H\mathbf{\Phi} + \alpha k\,\mathrm{diag}\left\{\left[((a)_1^2+\epsilon)^{k/2-1} \;\; \cdots \;\; ((a)_M^2+\epsilon)^{k/2-1}\right]\right\}$, the gradient $\nabla J_\epsilon(\mathbf{a}) = -2\mathbf{\Phi}^H\mathbf{r} + \mathbf{H}(\mathbf{a})\mathbf{a}$, leading to the quasi-Newton iteration $\mathbf{a}^{(n+1)} = \mathbf{H}^{-1}\left(\mathbf{a}^{(n)}\right)2\mathbf{\Phi}^H\mathbf{r}$ (Çetin and Karl, 2001)

Greedy Graph-Structured Algorithm

- with $\mathbf{\Phi}$ having $\mathcal{O}(N^2P)$ columns, the quasi-Newton method is expensive in memory and computation
- the main idea of the graph-structured algorithm is to consider a subset of basis vectors from a subgraph, called the guiding graph, at a time and iteratively move the guiding graph around within the basis graph in search of the true anisotropy
- the diagram below illustrates the iterations of a search where true anisotropy is represented by the node with the X



Strategy: Guided Depth-First Search

- follow one path down starting from the root with the path based on a heuristic; if goal not found on first pass down, back-track, also based on a heuristic
- on each search iteration, $\mathbf{r} = \mathbf{\Phi}^{(i)}\mathbf{a}^{(i)}$ is solved using the quasi-Newton method, where $\mathbf{\Phi}^{(i)}$ contains basis elements from P guiding graphs

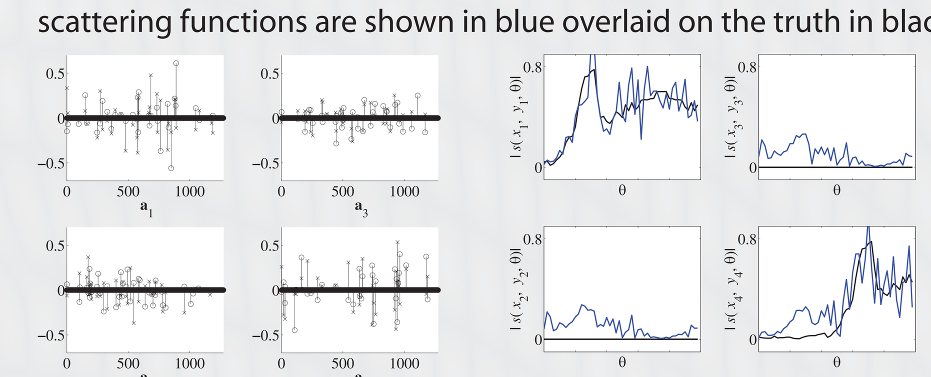
Heuristics and Stopping Criteria

- true anisotropy finer than current guiding graph:
  - bottom row coefficients non-zero
  - slide guiding graph down
  - guide left or right: weighted average of bottom row coefficients
- true anisotropy coarser than current guiding graph:
  - top row coefficient non-zero
  - slide guiding graph up
- true anisotropy inside current guiding graph:
  - true coefficients non-zero
  - do not move guiding graph – stop
- the above heuristic and stopping criterion is for each of the P guiding graphs individually
- there are P simultaneous searches, but the searches are coupled because spatial locations interact within one system of equations

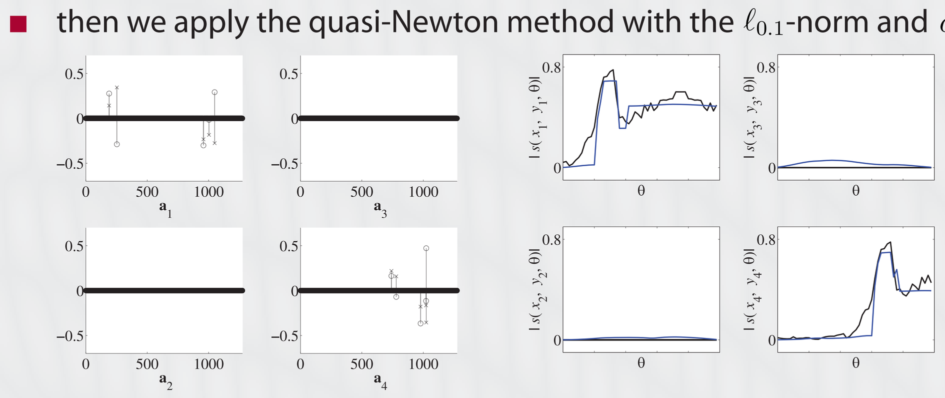
Examples

Quasi-Newton Method — XPatch Data

- there are $P = 4$ spatial locations and $N = 50$ angles over a 98° aperture
- first we obtain the least-squares solution as a baseline for comparison
- the $M = 1275$ coefficients of the solution are shown as a stem plot for each spatial location with real part ° and imaginary part ˣ; the corresponding scattering functions are shown in blue overlaid on the truth in black



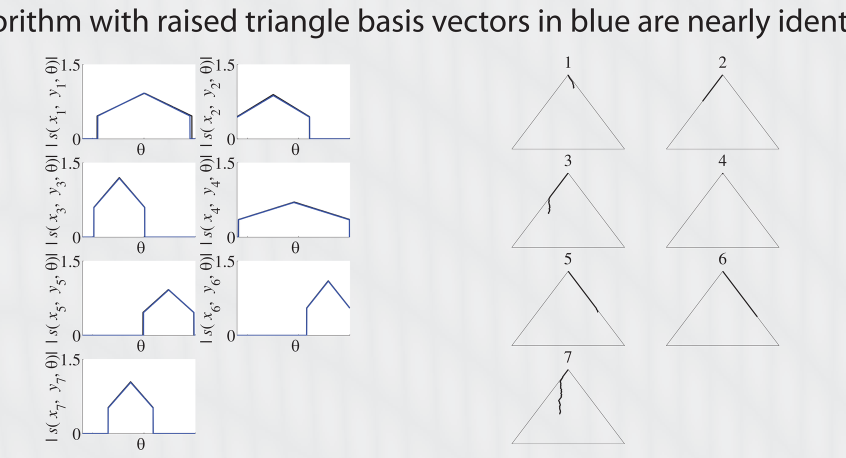
- then we apply the quasi-Newton method with the $\ell_{0.1}$-norm and $\alpha = 1$



- the quasi-Newton solution is sparser in the coefficients and importantly explains the underlying truth better

Graph-Structured Algorithm — Synthetic Data

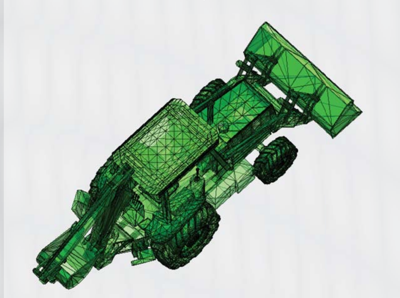
- there are $P = 7$ spatial locations and $N = 1541$ angles over a 110° aperture
- the true anisotropy in black and the solution from the graph-structured algorithm with raised triangle basis vectors in blue are nearly identical



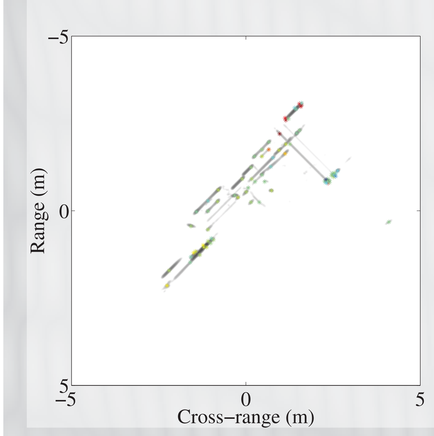
- shown above to the right, the paths of the guiding graphs for each spatial location reach the region of the basis graph of true anisotropy fairly directly

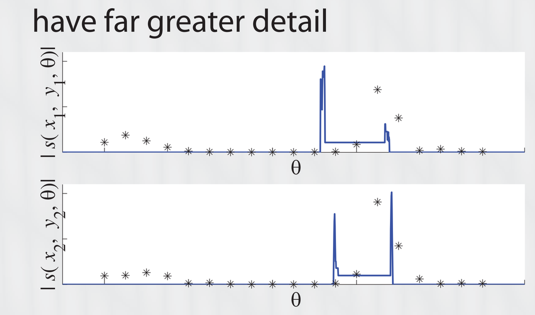
Graph-Structured Algorithm — Backhoe Data

- there are $P = 75$ spatial locations and $N = 1541$ angles over a 110° aperture
- anisotropy is characterized for a selected set of 75 spatial locations



- the bottom left image shows the center angle of the characterized anisotropy for each scatterer as a color – red is at one end of the range and blue is at the other end, with green in the middle (magnitude and persistence are not indicated); reds are apparent at one side of the front shovel of the backhoe and blues at the other side
- solutions at two of the spatial locations are shown in blue along with pixel values from subaperture analysis as asterisks; our formulation is able to recover solutions that have far greater detail





Presented at SPIE Defense and Security Symposium, Algorithms for Synthetic Aperture Radar Imagery XIII, April 17-20, 2006