TOOLS FOR ANALYZING SHAPES OF CURVES AND SURFACES

ANUJ SRIVASTAVA
Department of Statistics
Florida State University

MURI: INTGERATED FUSION AND SENSOR MANAGEMENT FOR ATE

Innovative Front-End Processing

1c. Statistical shape theory

- Characterize objects in signals and images.

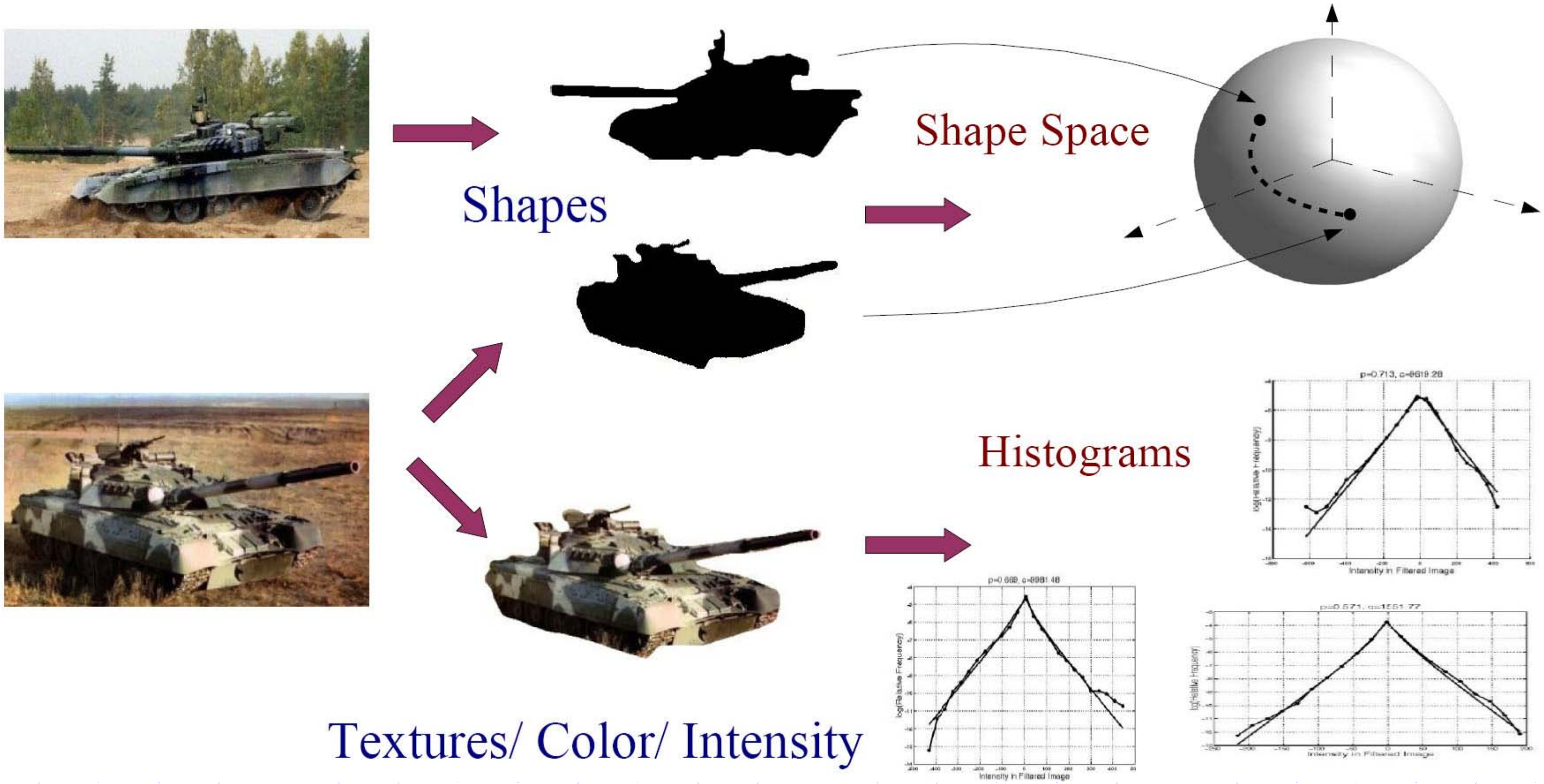
- Seek physics-based features for robust, information-based fusion.

- An important feature seems to be SHAPE. Human visual system relies heavily on edges, contours, and analysis of their shapes.

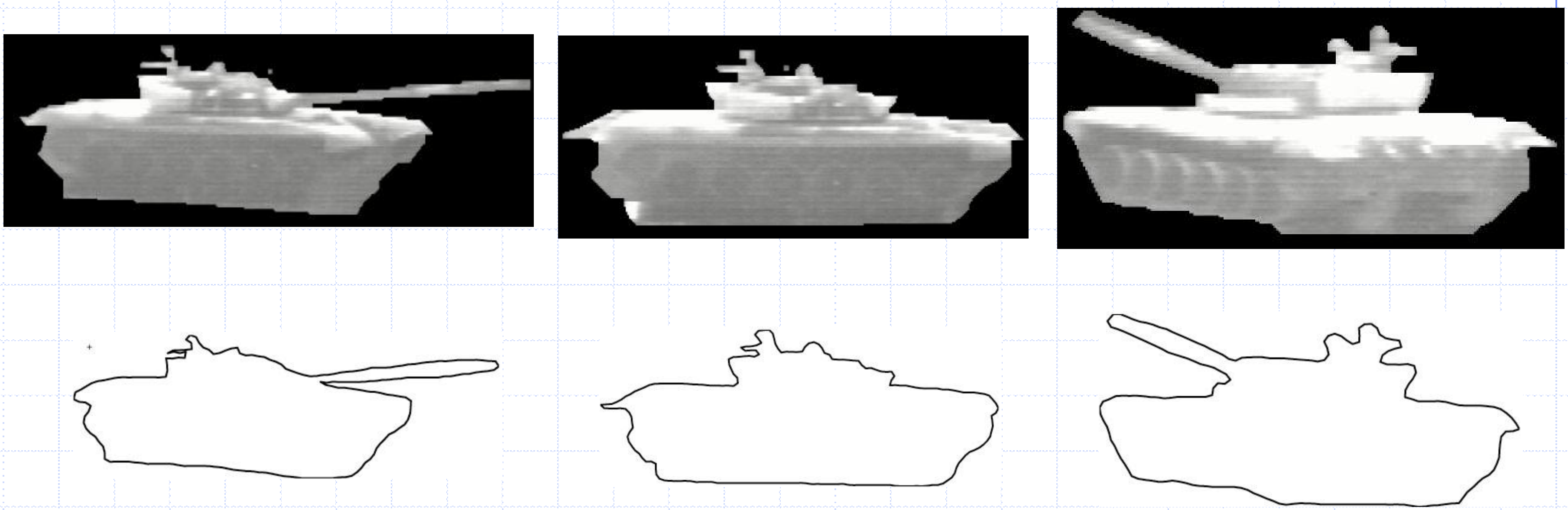
- For 2D images, boundaries are curves, while for 3D objects boundaries are surfaces.

TWO DISTINCT TARGET ATTRIBUTES

SHAPES & TEXTURE



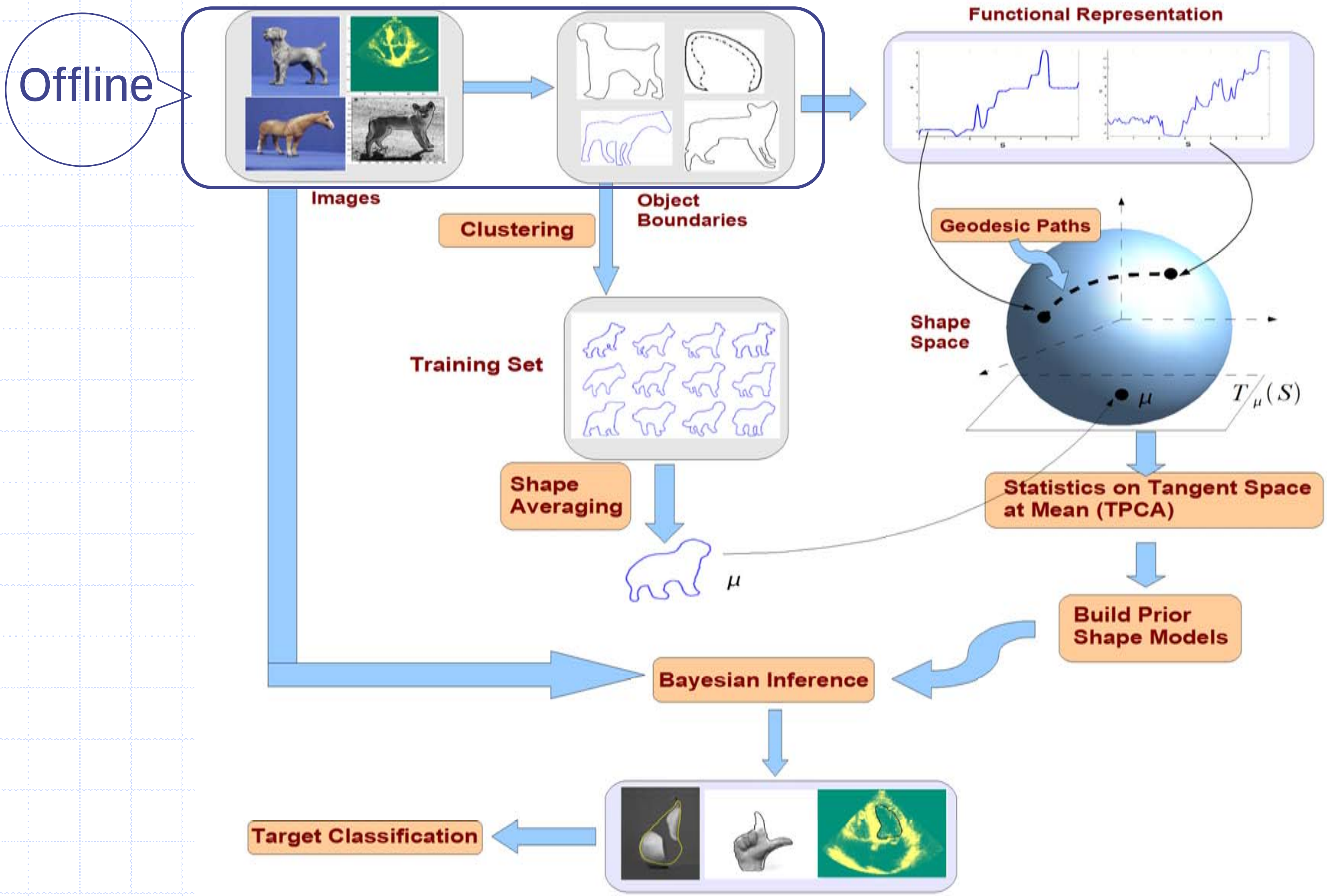
SHAPE ANALYSIS FOR ATE



Shape provides a partial characterization of objects

Disclaimer: shape analysis is generally not useful in far-field targets when there are very few pixels on targets.

OVERVIEW OF SHAPE ANALYSIS



PAST RESEARCH (2002-2006)

Shape Analysis of Closed Curves

Unit-Speed Curves: Bending Only

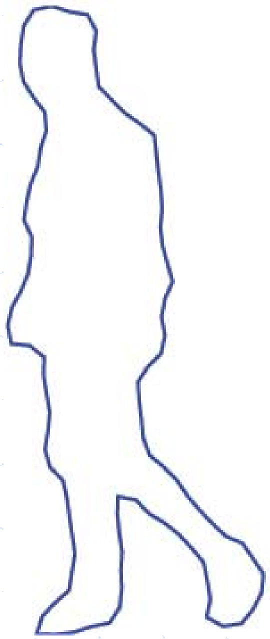
- Represent a curve by its angle function
- Use a bending metric to compute geodesics using a shooting method (IEEE PAMI, 2004)
- Compute statistics using TPCA method (IEEE PAMI, 2005)

Arbitrary-Speed Curves: Elastic Shapes

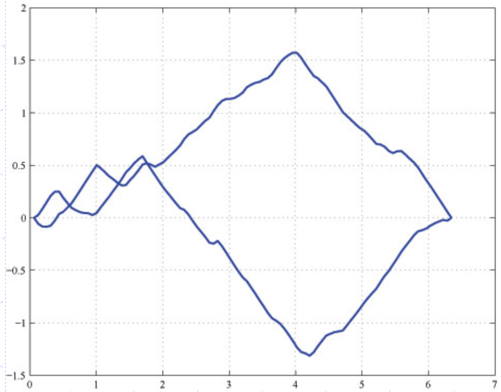
- Represent a curve by its angle and speed functions
- Use a bending metric to compute geodesics using a shooting method (CVPR, 2004; IJCV, 2007)
- Compute statistics using TPCA method (ACCV, 2006)

Others: Michor-Mumford (2004, 05), Younes (1998), Yezzi-Mennucci (2005)

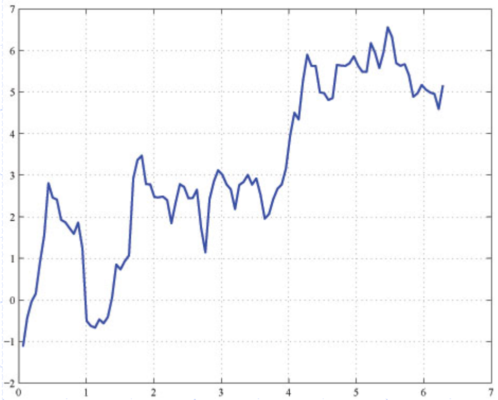
REPRESENTATION OF A CURVE



If the curve is arc-length parameterized:



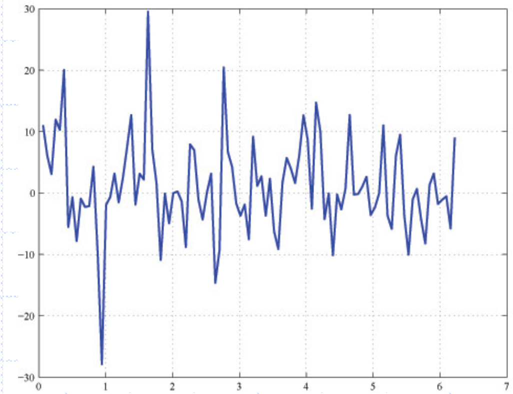
x, y coordinates



angle function

1^{st} derivative

(velocity function)



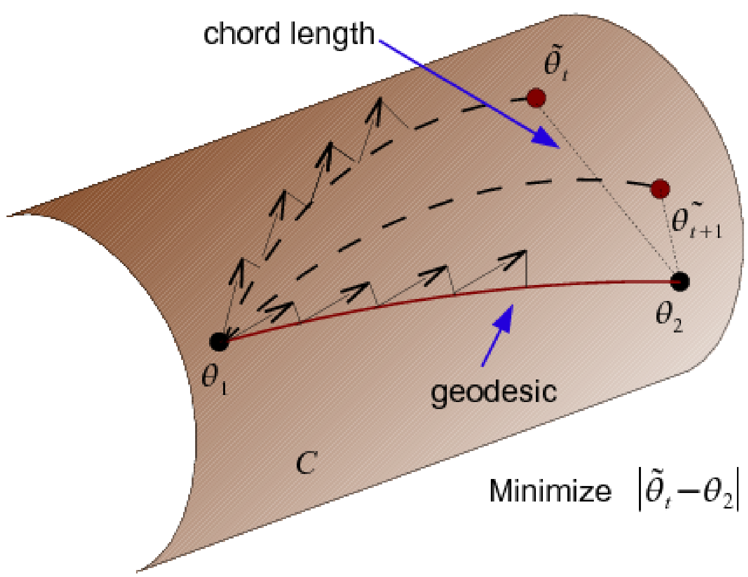
curvature function

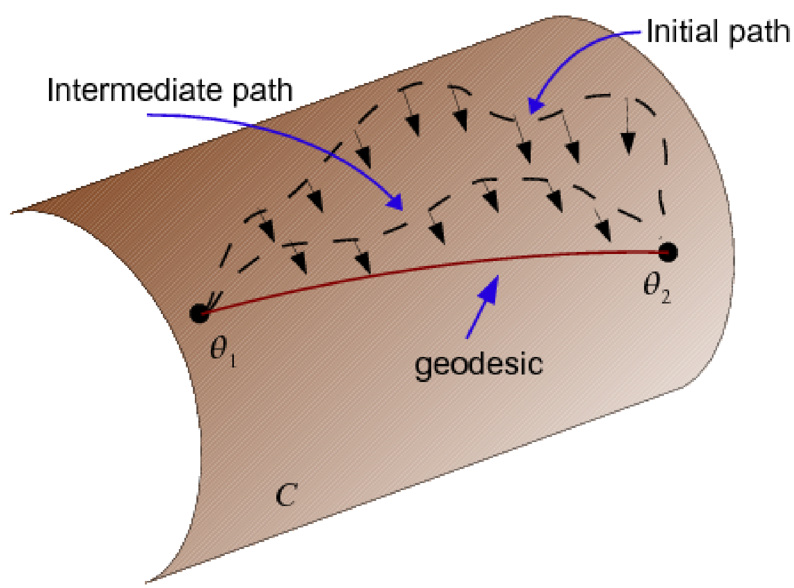
2^{nd} derivative

(torsion, curvature)

GEODESICS ON SHAPE SPACES

Geodesics are computed using numerical techniques, analytical solutions are not available

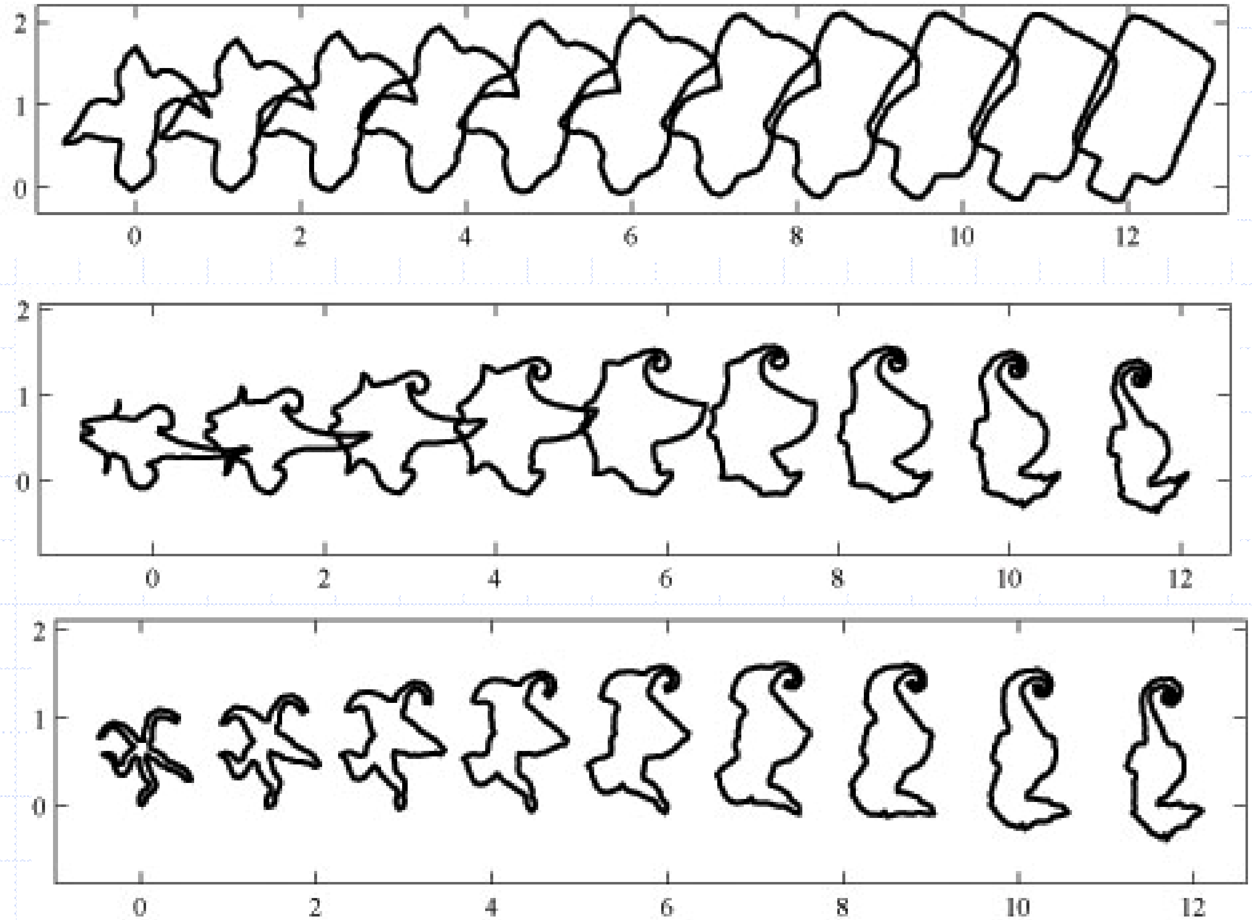




Shooting Method

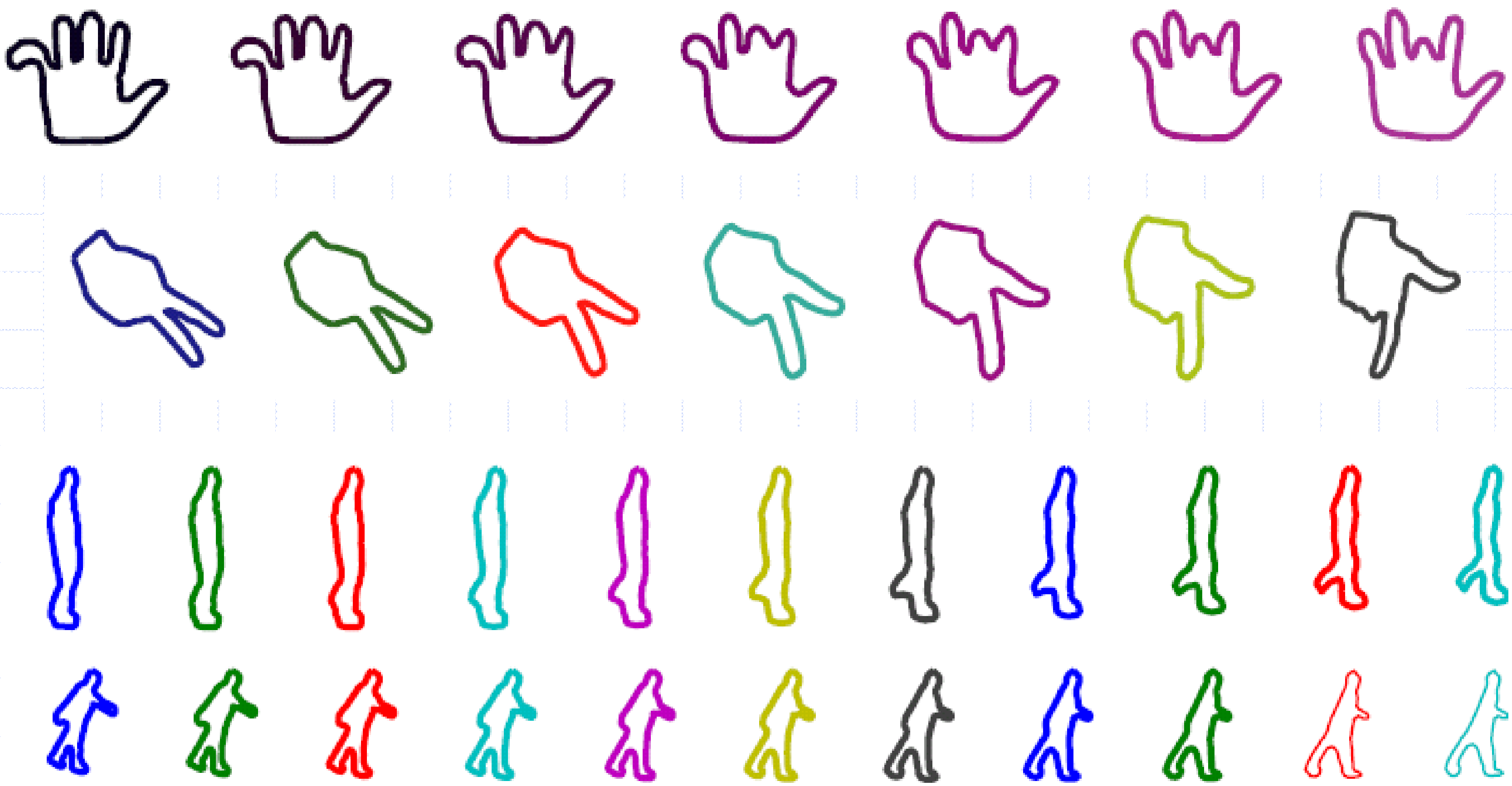
Path Straightening

NON-ELASTIC GEODESICS ON SHAPE SPACES



Fully automatic, no user input needed

ELASTIC GEODESICS ON SHAPE SPACES



David Kaziska, Human recognition by modeling gait as a stochastic process on shape space, PhD thesis, 2006

FIRST YEAR RESEARCH

Three main areas of research:

1. Improving past methods for shape analysis.
2. Use of shape priors in estimation of boundaries in images.
3. Classification of objects using sampled points.

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METRICS FOR INFORMATION GEOMETRY

- Space of probability density functions:

$$\mathcal{F} = \{f : [0,1] \mapsto \mathbb{R}_+ | \int_0^1 f(s)ds = 1\}$$

- Fisher-Rao metric: for $v_1, v_2 \in T_f(\mathcal{F})$

$$\langle v_1, v_2 \rangle = \int_x v_1(x) v_2(x) \frac{1}{f(x)} \; dx$$

Square-root Representation: (Bhattacharya 1943, Rao 1945)

$$\Psi = \{\psi : [0,1] \mapsto \mathbb{R} | \int \psi(x)^2 dx = 1\}$$

Fisher-Rao metric: for $v_1, v_2 \in T_\psi(\Psi)$

$$\boxed{\langle v_1, v_2 \rangle = \int_x v_1(x) v_2(x) \; dx}$$

Srivastava et al., *Riemannian Analysis of Probability Density Functions with Applications in Computer Vision*, CVPR 2007

SHAPE ANALYSIS: IMPROVING EFFICIENCY

Parametrized Curve: $\beta(s)$, Velocity Vector $\dot{\beta}(s)$

Write $\dot{\beta}(s) = |\dot{\beta}(s)| e^{i\theta(s)} = e^{\phi(s)} e^{i\theta(s)}$

Old representation: (ϕ, θ)

New representation: $q(s)$

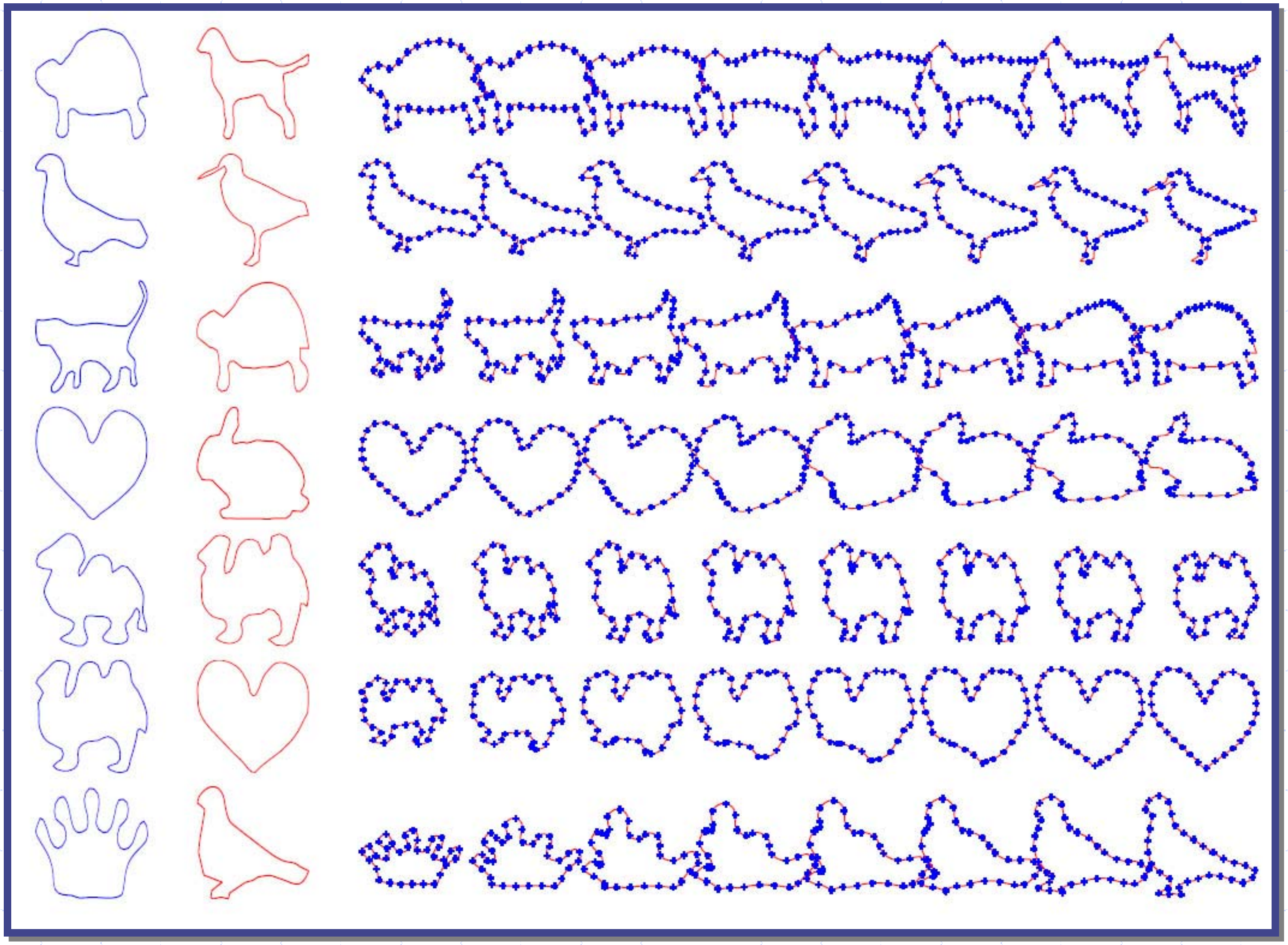
$$\text{Define } q(s) = \sqrt{|\dot{\beta}(s)|} e^{i\theta(s)} \in \mathbb{R}^2$$

$$\text{Define } q(s) = \frac{\dot{\beta}(s)}{\sqrt{|\dot{\beta}(s)|}} \in \mathbb{R}^n$$

Several computations simplify. In particular, the elastic metric simplifies to become L^2 metric.

Joshi et al., *An Efficient Representation for Computing Geodesics Between n-Dimensional Elastic Curves*, CVPR 2007

SQUARE-ROOT ELASTIC (SRE) FRAMEWORK



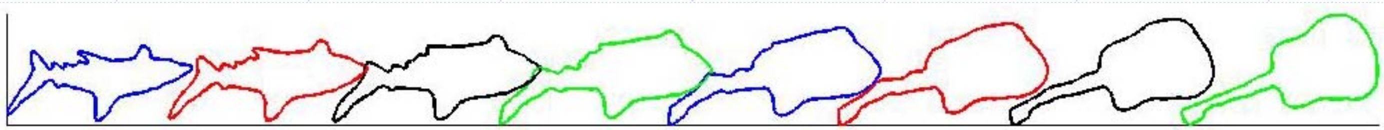
PROGRESS IN SHAPE ANALYSIS OF CURVES

Representation	Domain	Metric	Geodesic Comp.
Angle or Curvature	$\mathbb{R}^2$	Bending $\mathbb{L}^2$	Shooting Method
(Log-Speed, Angle)	$\mathbb{R}^2$	Elastic	Shooting Method
Square-Root Velocity	$\mathbb{R}^n$	Elastic $\mathbb{L}^2$	Path-Straightening Method

EXTENSION TO

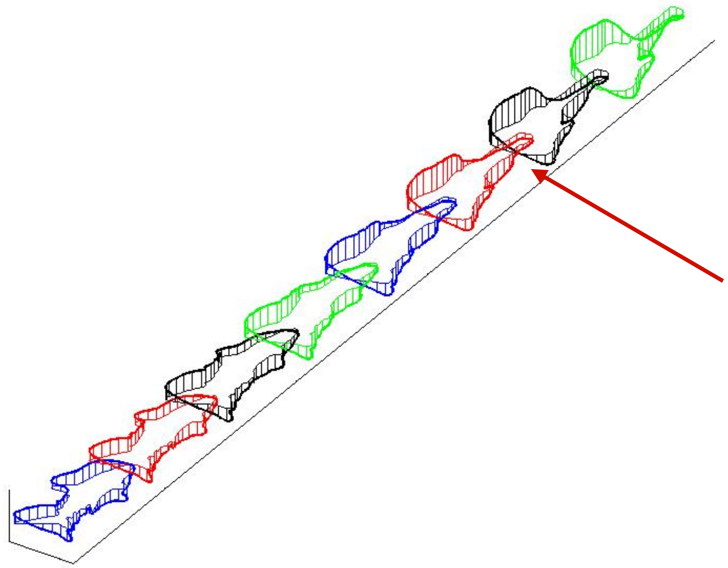
JOINT SHAPE & TEXTURE ANALYSIS

Shape Only: Geodesic between planar shapes



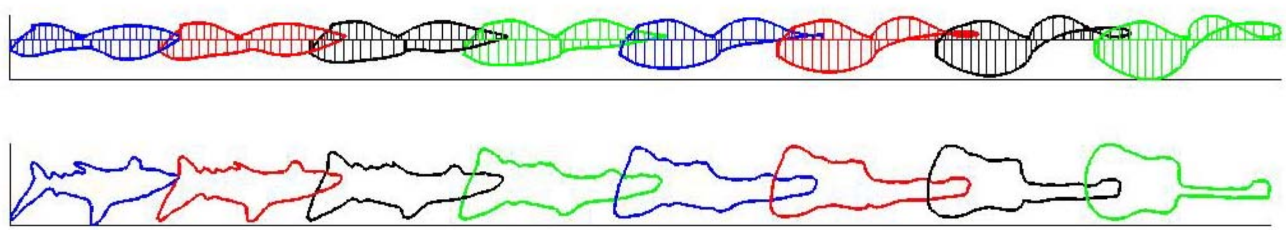
JOINT SHAPE & TEXTURE ANALYSIS

Shape & Texture: Use texture as additional coordinates



3D geodesic

2D Projections



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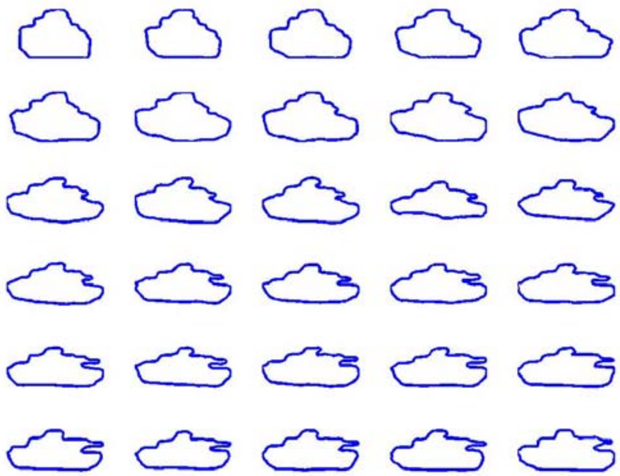
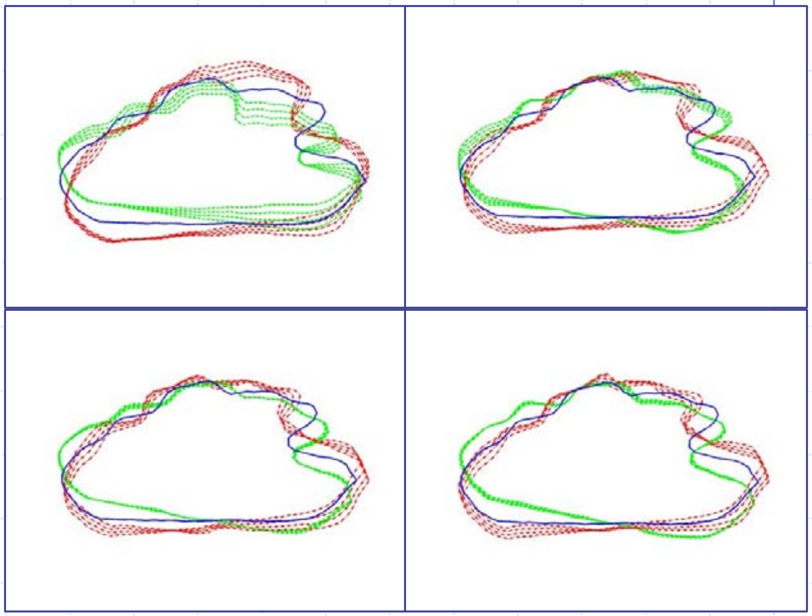
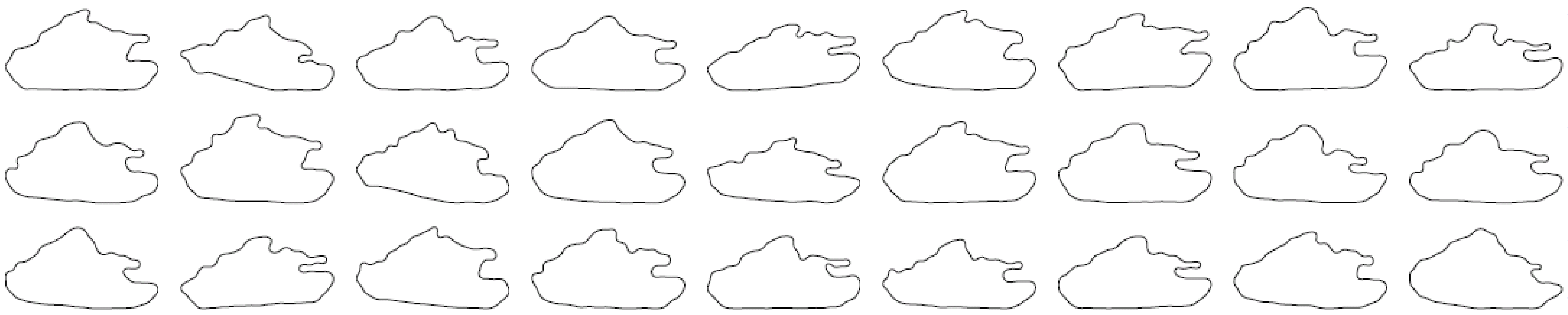
STATISTICAL SUMMARIES OF SHAPES



Mean Shape



Eigen modes of shape variations



Random samples from a model learnt from data

IMPROVING CONTOUR EXTRACTION

Traditional algorithms for contour extraction use active contours

Contours are driven by PDEs, based on gradient of two energy terms:

$$E_{total} = E_{image} + E_{smooth}$$

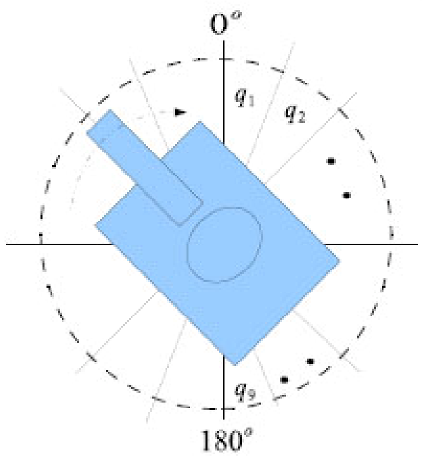
E_{image} relates the contour to the image

E_{smooth} forces the contour to remain smooth

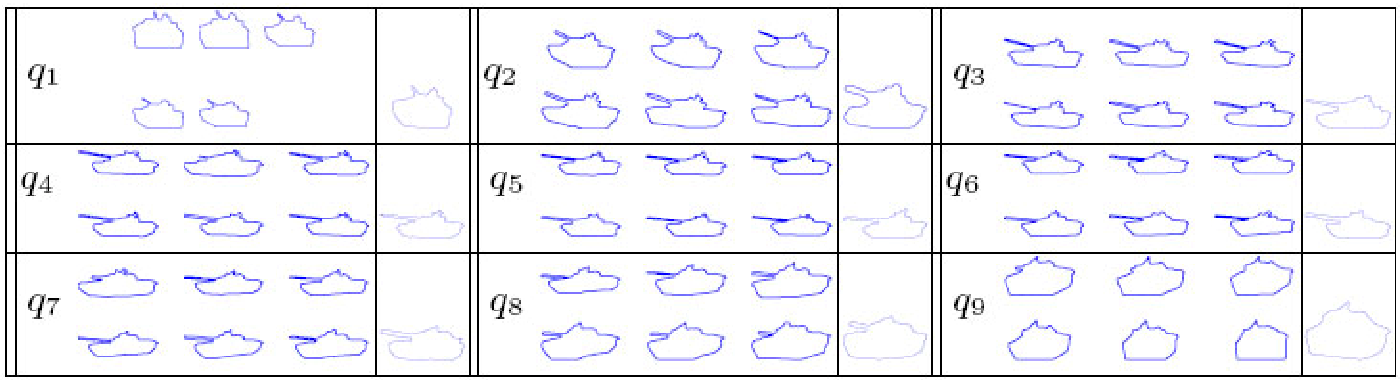
We add a third term which incorporates our prior knowledge about possible shapes

E_{prior} favors a certain shape class, e.g tanks

PRIOR SHAPE MODELS FOR TARGETS



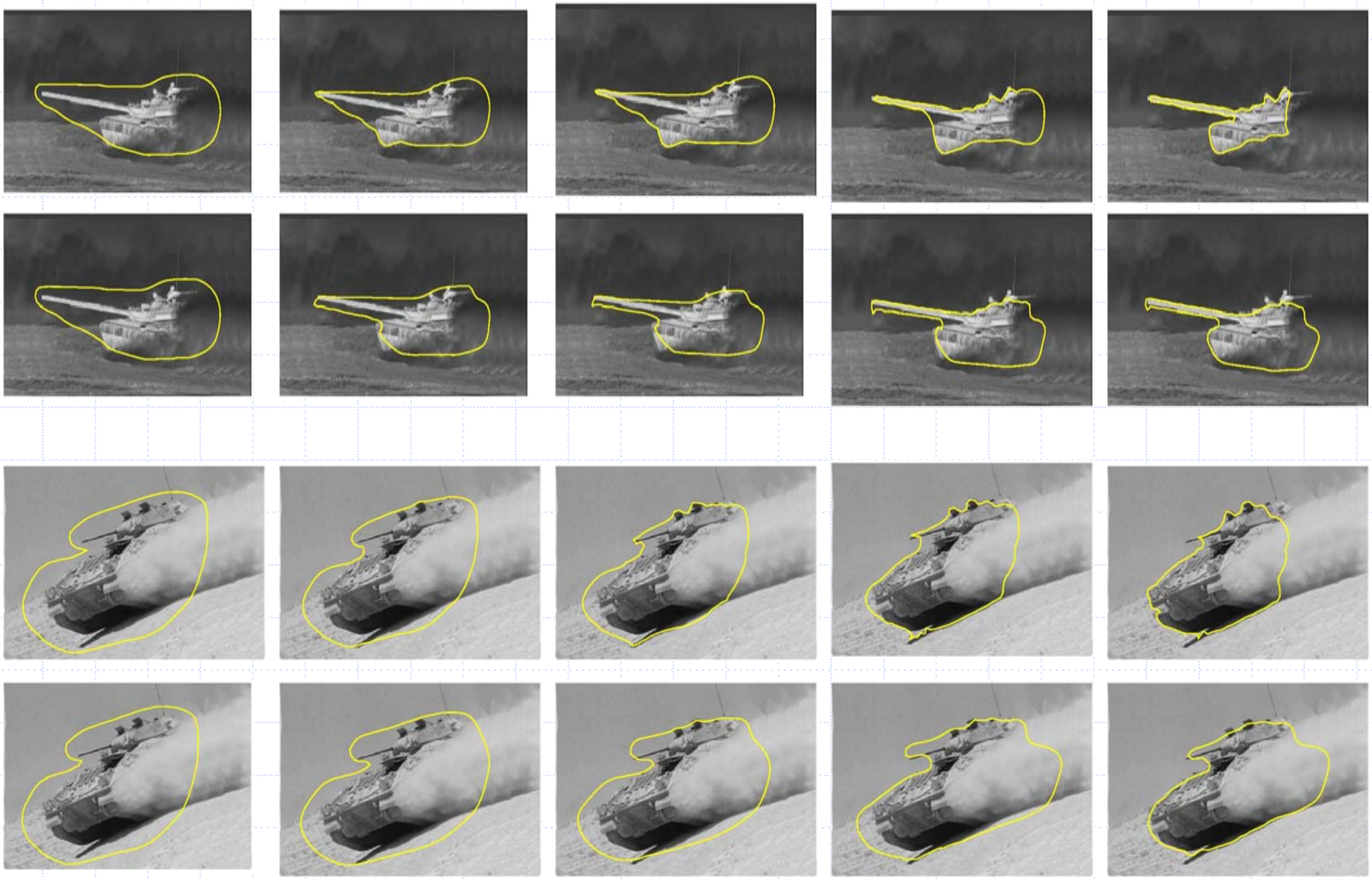
Tank: Image capture setup



Learning shape models for different sectors

Data courtesy Dr. Richard Sims, US AMCOM, Huntsville.

IMPROVING CONTOUR EXTRACTION



Joshi and Srivastava, Bayesian Active Contours, IJCV, in review

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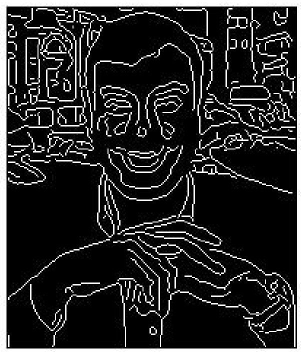
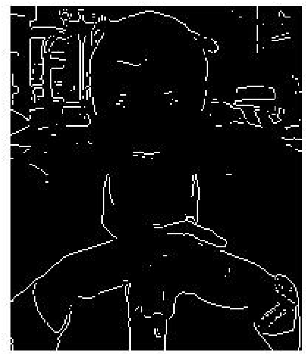
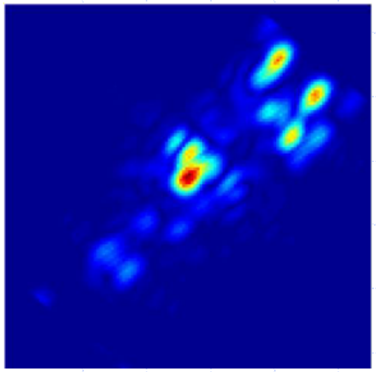
CLASSIFICATION OF OBJECTS FROM A SET OF PRIMITIVES – POINTS, EDGES

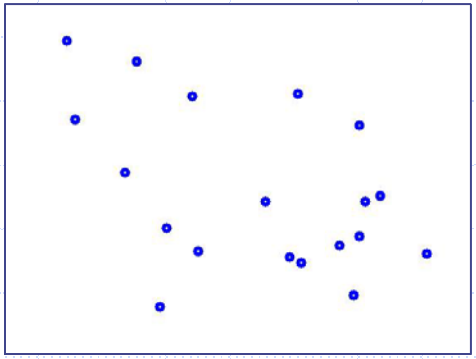
Backhoe – sparse reconstruction







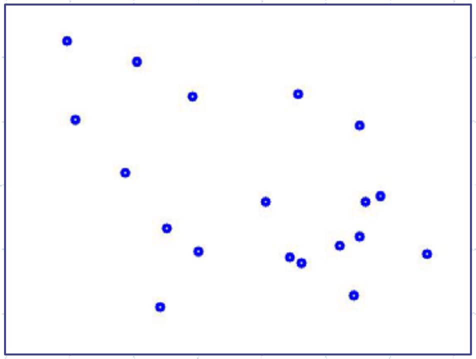
Low-level processing results in points, edges, curvelets, junctions, etc



“Connecting the dots”

CLASSIFICATION OF ORDERED POINT SETS

Problem: Given an ordered set of points, classify them into one of given shape classes



Classes: bone, bird, bottle, brick, camel, cat, carriage, car, chopper, crown, ...

Courtesy: Kimia Database

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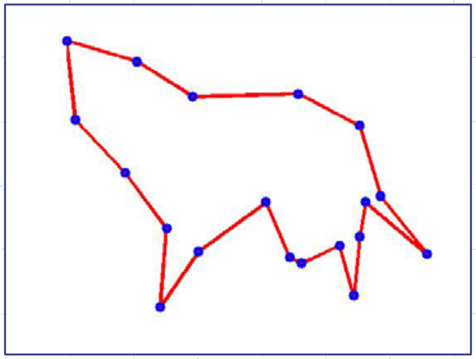
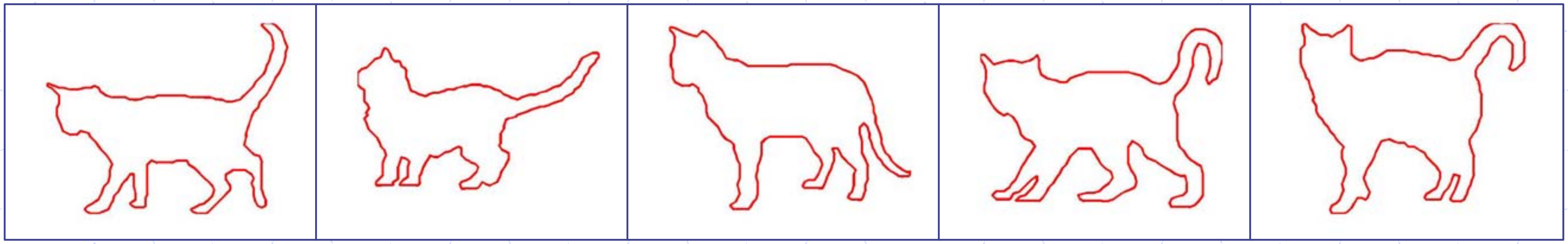


Classes: bone, bird, bottle, brick, camel, cat, carriage, car, chopper, crown, ...

Knowledge Base: Past Observations (continuous curves)

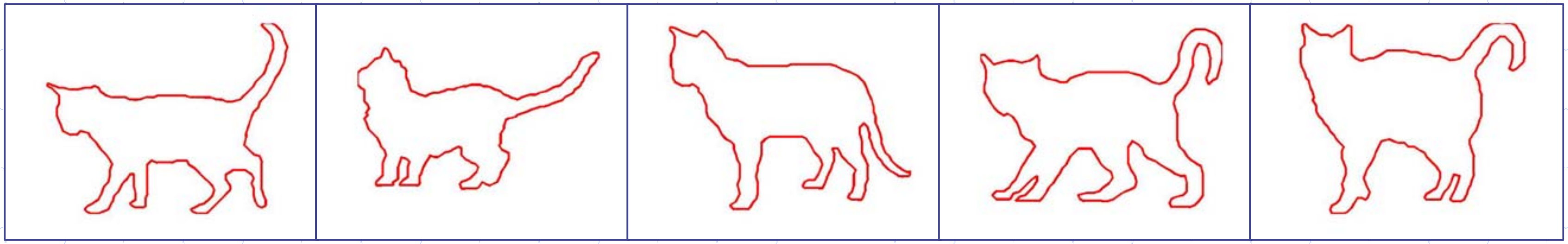


Courtesy: Kimia Database

MODELING VARIABILITY IN OBSERVATIONS

Three Sources of Variability

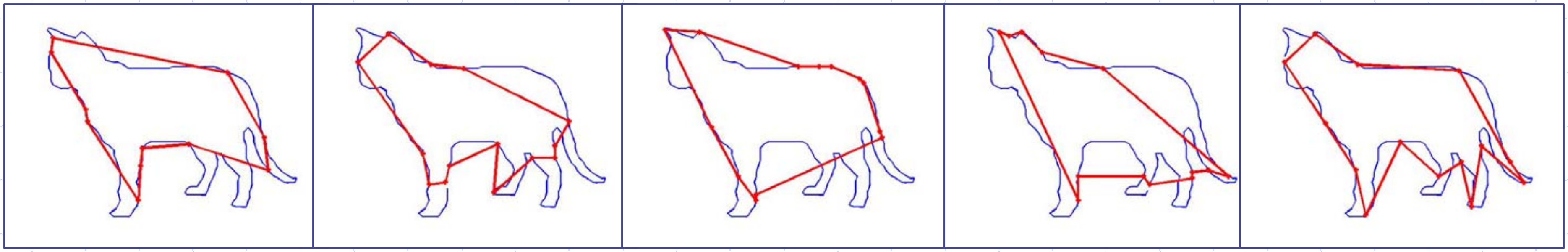
1. Variability in shapes within a class

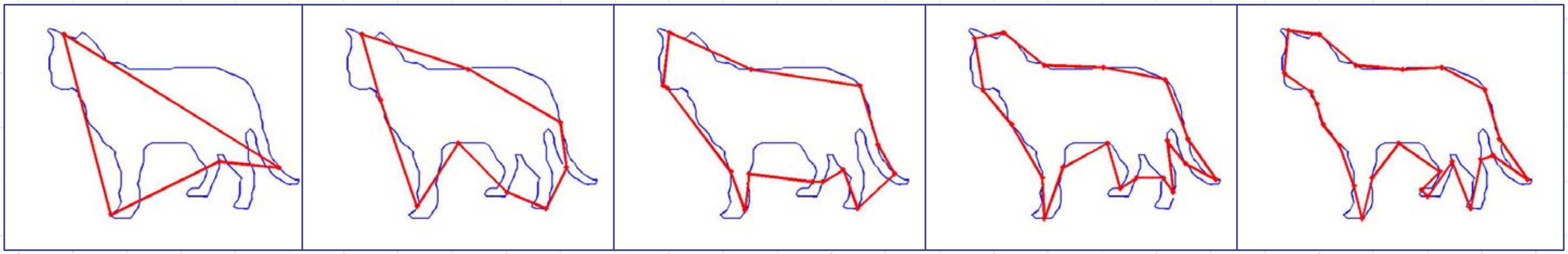


MODELING VARIABILITY IN OBSERVATIONS

Three Sources of Variability

2. Variability in sampling a curve

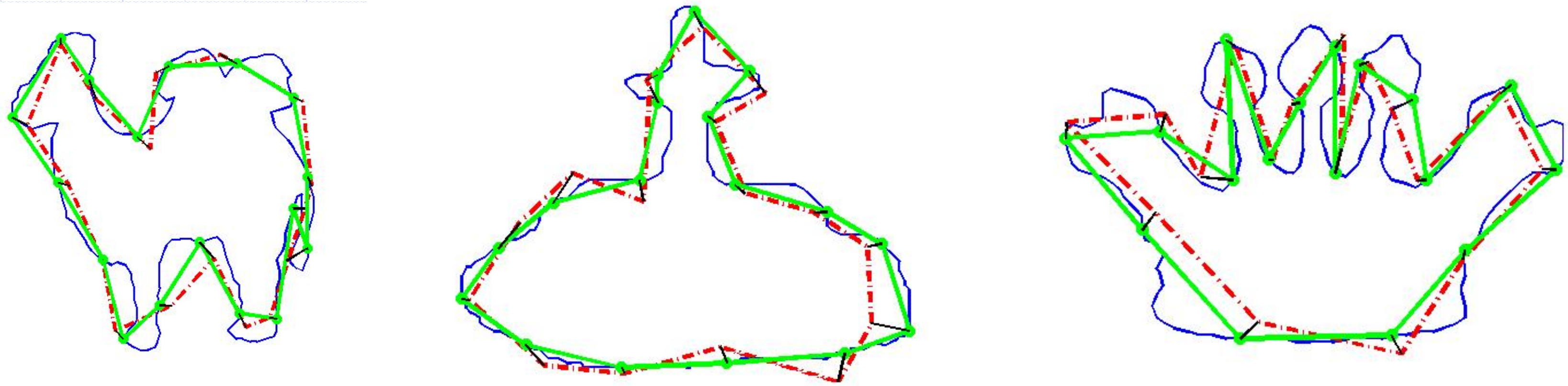




MODELING VARIABILITY IN OBSERVATIONS

Three Sources of Variability

3. Observation Noise



Observation Model:

$$\mathbf{y} = \beta(\gamma(U)) + w$$

Provides the likelihood function $P(\mathbf{y}|\beta, \gamma)$

BAYESIAN CLASSIFICATION

MAP Estimation of Shape Class

$$\hat{C} = \mathrm{argmax}_{C_i} \, P(C_i|\mathbf{y})$$

Posterior Probability

shape

$$P(C_i|\mathbf{y}) = \frac{P_0(C_i)}{P(\mathbf{y})} \int\int\int P(\mathbf{y}|q,x,\gamma) P(q|C_i) P(x|C_i) P(\gamma|C_i) dq \; dx \; d\gamma$$

pose

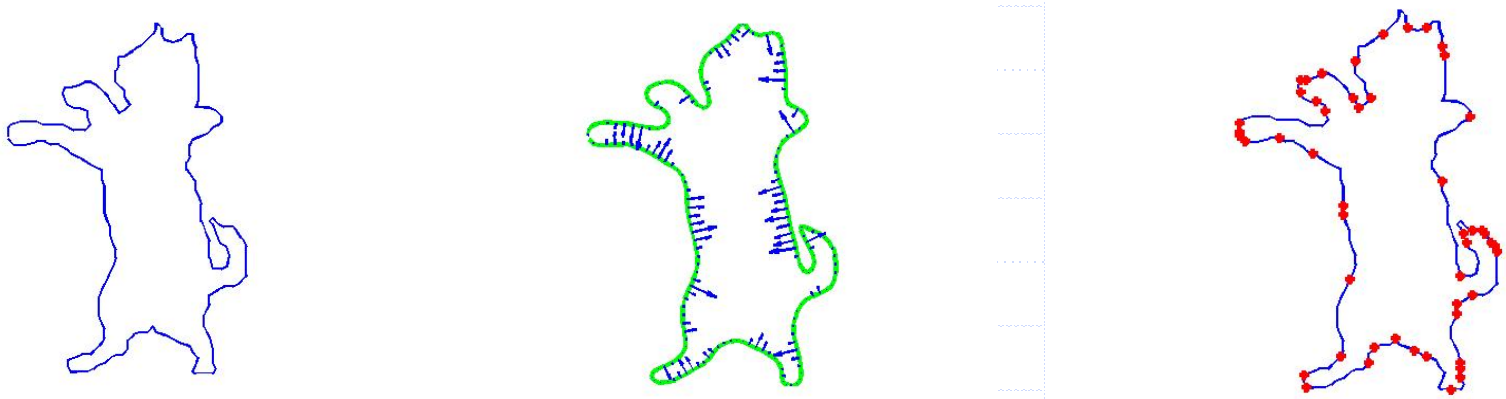
Class specific prior on sampling functions

A PRIOR MODEL ON SAMPLING FUNCTION

For a curve β, let κ be its curvature function

We prefer a sampling function that is inversely proportional to exponential of κ

$$\gamma : [0,1] \mapsto [0,1], \quad \gamma(t) = \frac{\int_0^t e^{\frac{-|\kappa(s)|}{\rho}} ds}{\int_0^1 e^{\frac{-|\kappa(s)|}{\rho}} ds}$$

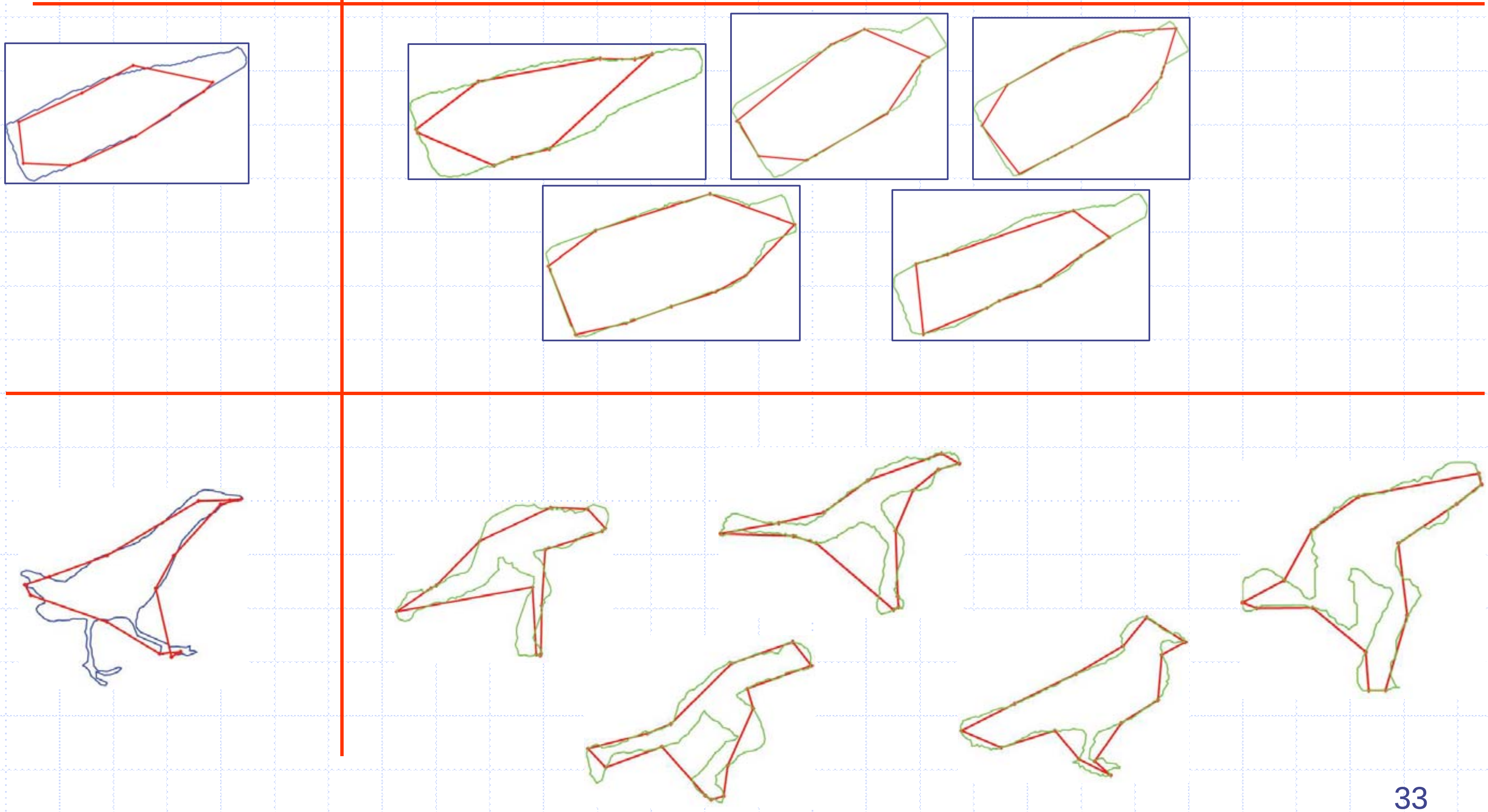


For each training shape, we can compute a sampling function

HIGH POSTERIOR PROBABILITY SAMPLES

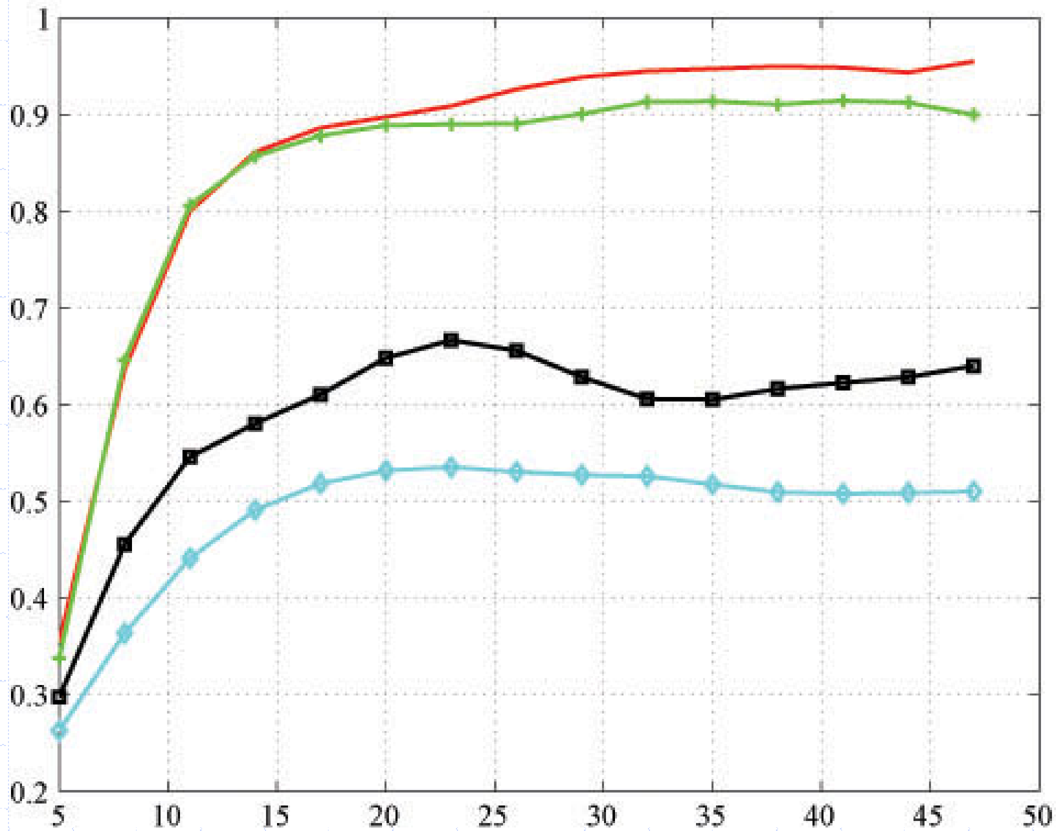
$$P(C_i|\mathbf{y}) = \frac{P_0(C_i)}{P(\mathbf{y})} \int\int\int P(\mathbf{y}|q,x,\gamma)P(q|C_i)P(x|C_i)P(\gamma|C_i)dq\ dx\ d\gamma$$

Data | High Probability Samples



CLASSIFICATION PERFORMANCE

Preliminary Results: Kimia Database, 17 shape classes

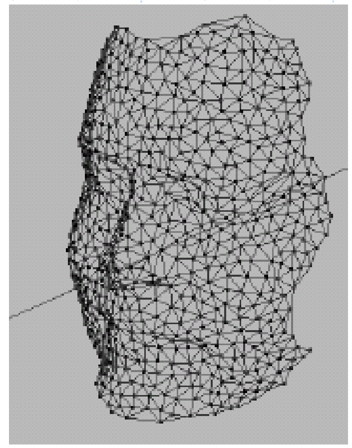
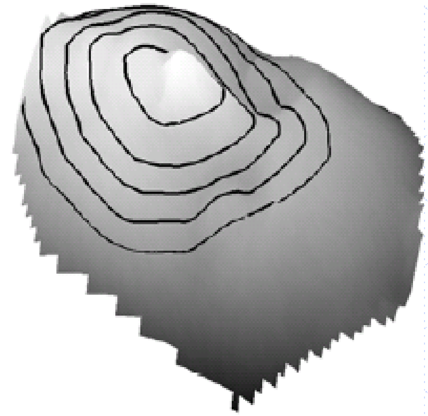
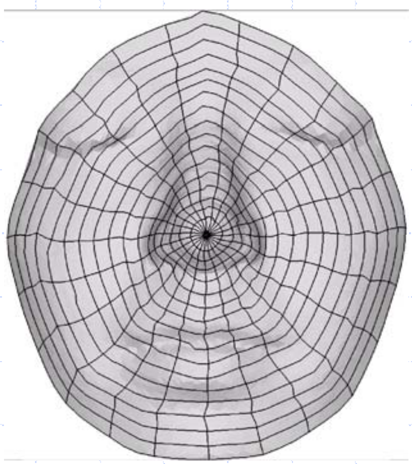
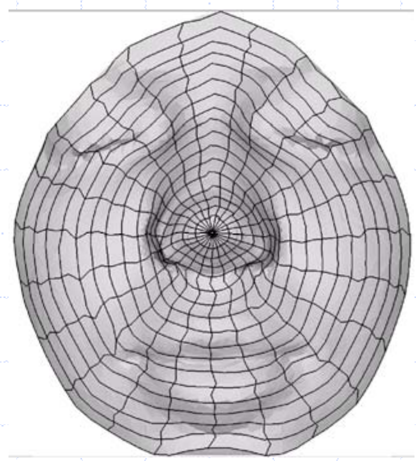
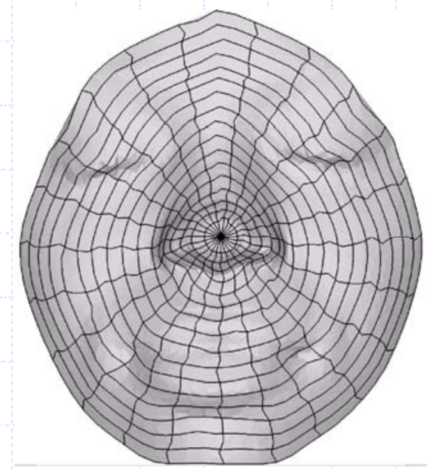


Recognition Rate Vs. Sample Size (different noise levels)

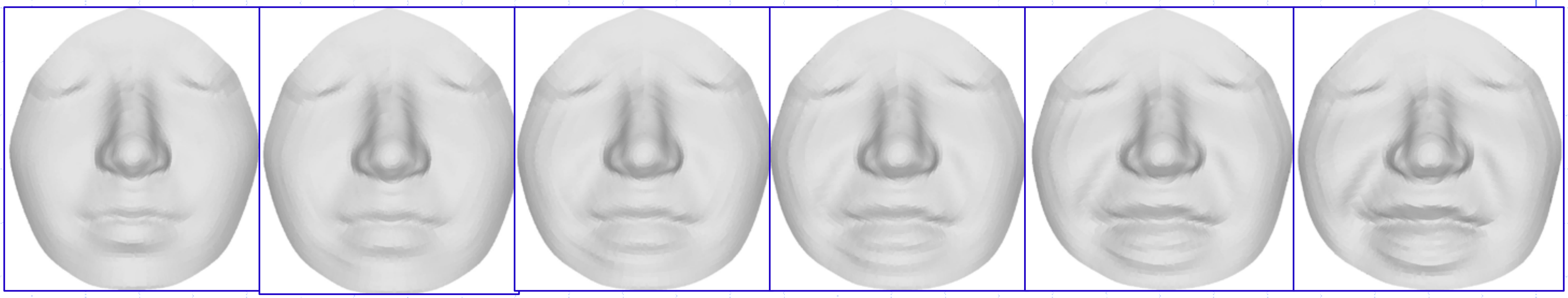
OTHER APPLICATIONS

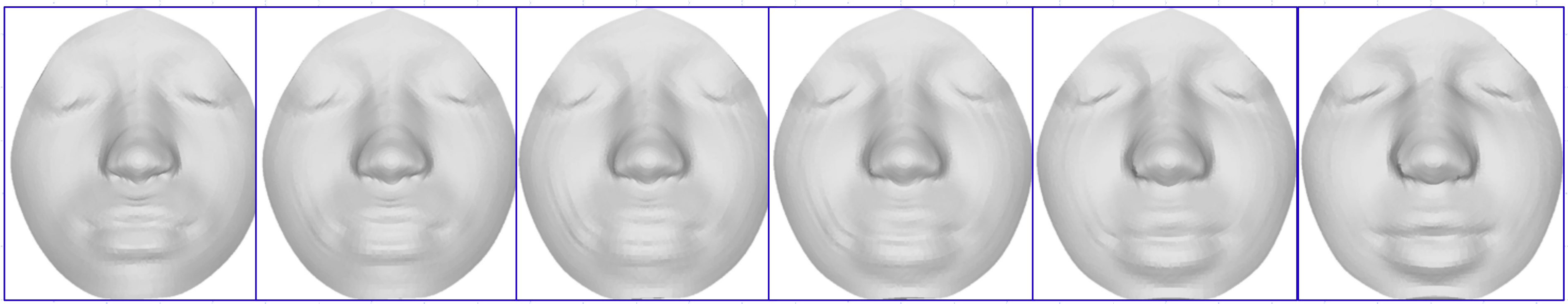
1. Face recognition by analyzing shapes of facial surfaces. (Collaboration with University of Lille; IEEE PAMI 2006, JMIV, in review 2007)
2. Studying shapes of neuronal fiber tracts in Human brain to separate schizophrenic and normal classes. (Collaboration with Vanderbilt U. VUIIS; EMMCVPR 2007)
3. Joint shape and texture analysis for classification of trees in aerial images (Collaboration with INRIA, Sophia-Antipolis; EUSIPCO 2007)
4. Shape/sampling models for human activity classification. (Collaboration with R. Chellappa's group at UMD)
5. Discussions with Nothrop-Grumman on ATR of underwater targets using airborne LIDAR imaging.

ANALYSIS OF 3D FACIAL SURFACES

GEODESIC PATHS BETWEEN FACIAL SURFACES





SECOND YEAR GOALS

1. One-Shot Learning of Shapes:

   Prediction and analysis of shapes from new perspectives.

2. Graphical Models for Studying Configurations of Shapes:

3. Joint Shape-Texture Analysis for Full Appearance Models.

ONE-SHOT LEARNING OF SHAPES

Setup:

From training data we already know the variability (distribution) of 2D shapes associated with a 3D object.

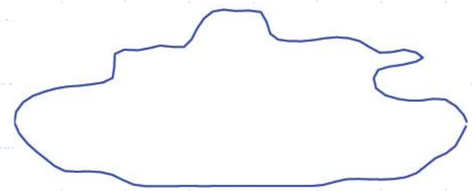
We obtain one image (shape) of a new object. What can we say about shape variability of this new object?

Using a well-known ("One-shot learning") approach, we can transfer the old distribution to new point. (Already done for pictures.

SUB-PROBLEM: SHAPE PREDICTION

Known Object:

M60



View 1

View 2

New Object:

T72

??

View 1

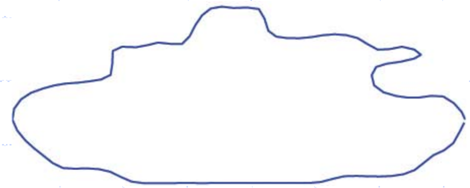
View 2

SUB-PROBLEM: SHAPE PREDICTION

Known Object:

M60

View 1



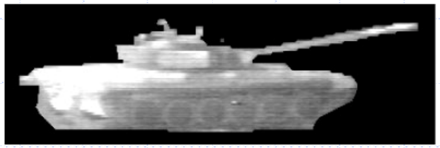
View 2

New Object:

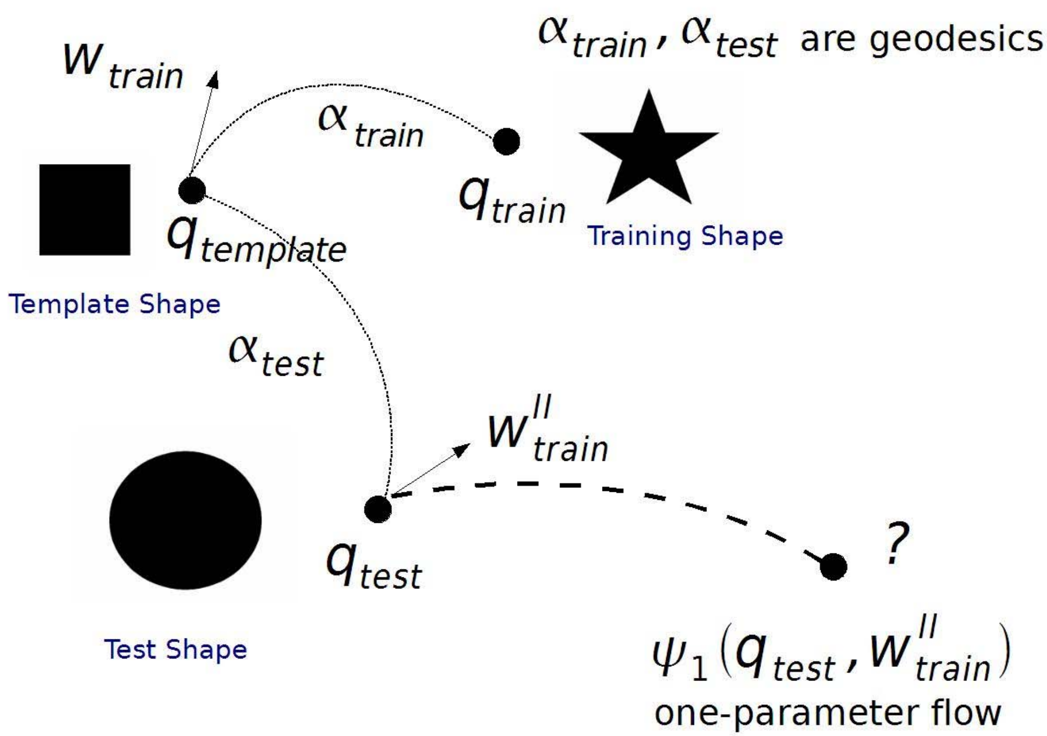
T72

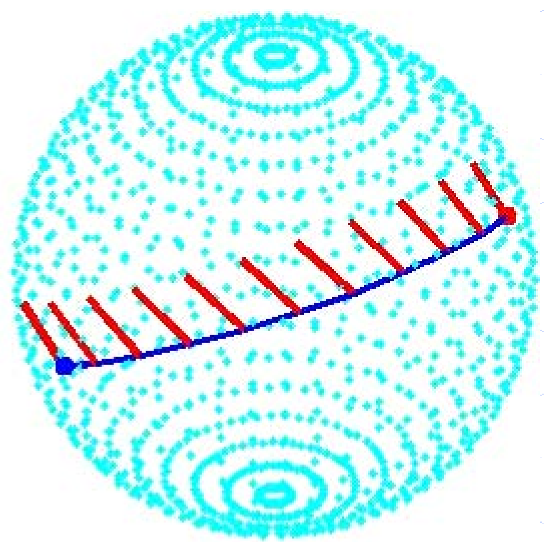
View 1

View 2

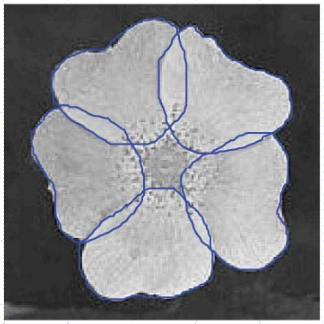


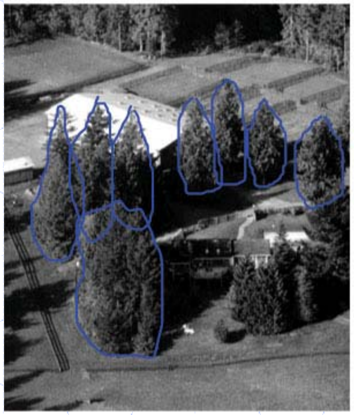
PARALLEL TRANSPORT OF VARIATIONS

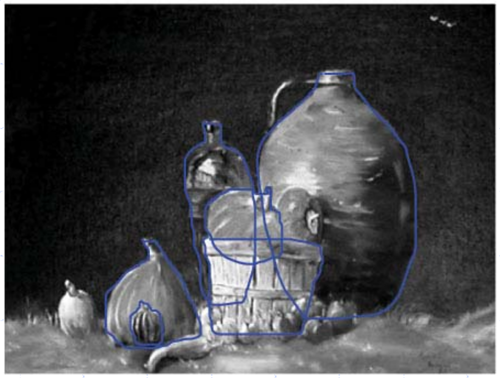




CONFIGURATIONS OF SHAPES

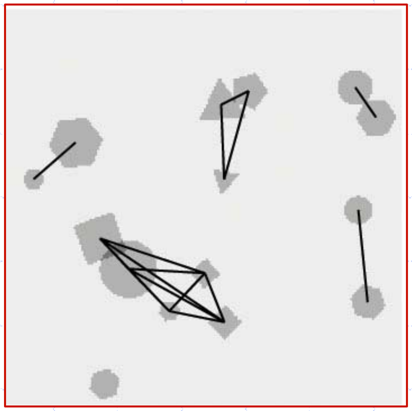




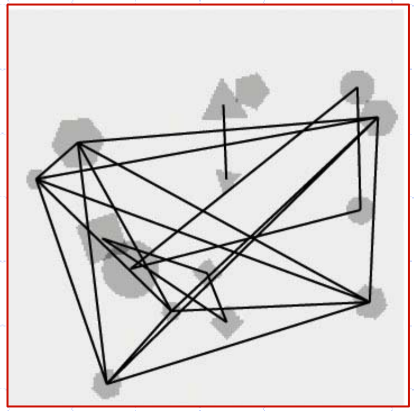


Multitudes of interacting shapes

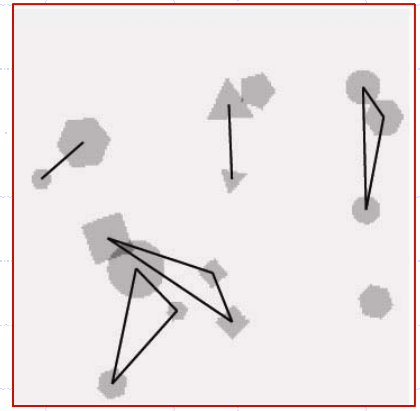
GRAPHICAL MODELS



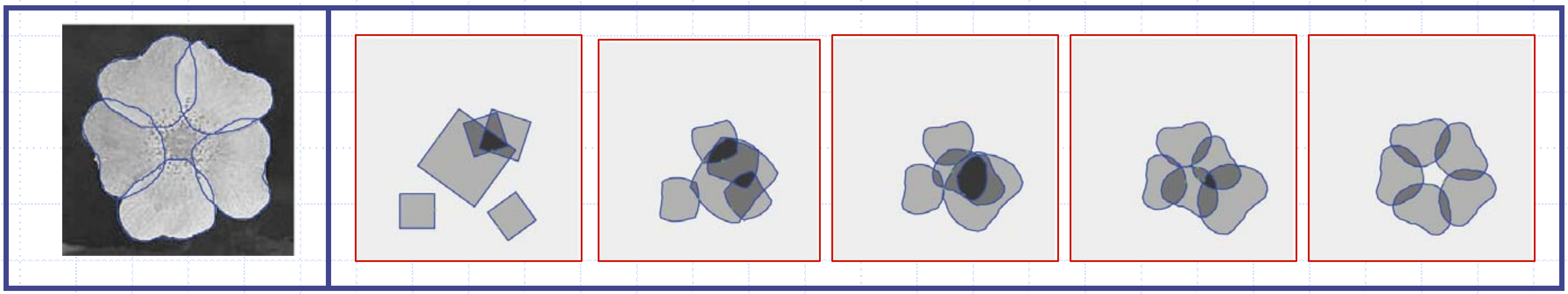
Position



Shape



Shape + Position



Real Config.

Energy-based evolution of shapes, pose →

SUMMARY

Three main items of research:

1. Improved past methods for shape analysis.
2. Used of shape priors in estimation of boundaries in images.
3. Developed classification of objects using sampled points.

Focus areas for next year:

- One-Shot Learning of Shapes
- Graphical Models for Studying Configurations of Shapes:
- Joint Shape-Texture Analysis for Full Appearance Models.

Srivastava, Klassen and Joshi, Statistical Analysis of Shapes of Curves, Springer Series in Statistics, In Preparation.

Comments, suggestions are most welcome!