Adaptive radar sensing strategies

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AFOSR MURI

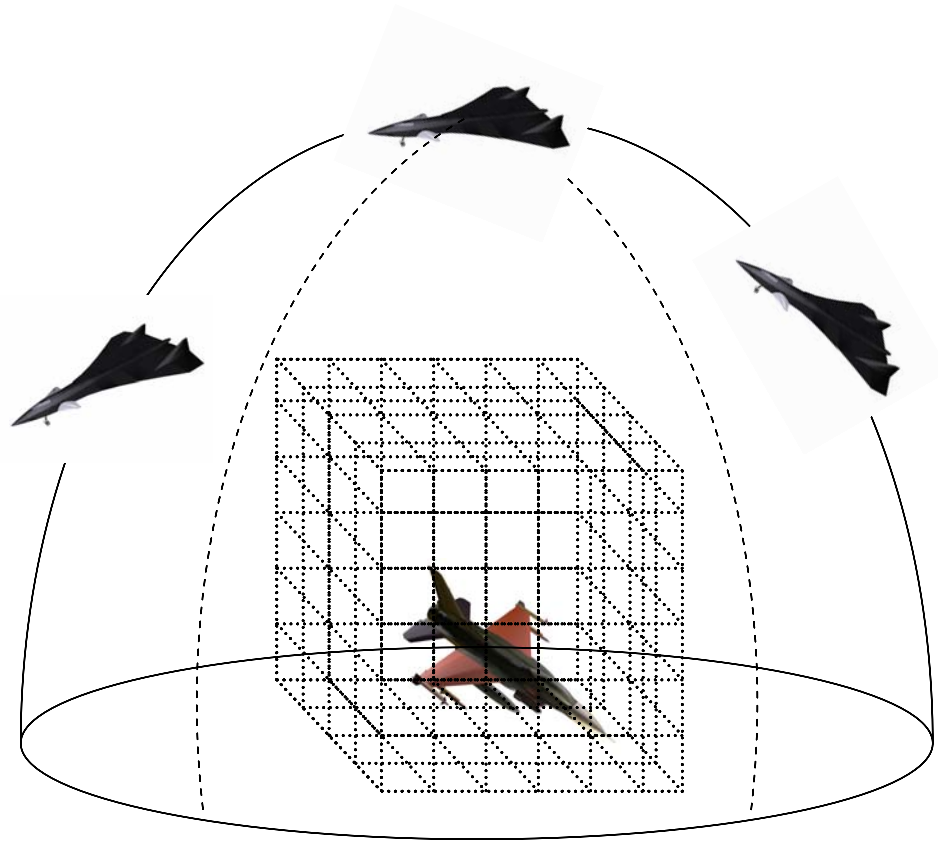
Integrated fusion, performance prediction,
and sensor management for ATE
(PI: R. Moses)

Outline

- ATE Vision and Research Approach
- Sequential Resource Allocation for ATE
  - Waveform design: optimizing spatial distribution
  - Waveform design: optimizing temporal distribution
- Second year directions
- Information items
  - Synergistic Activities
  - SM book to appear
  - Publications

I. ATE Vision and Research Approach

- ATE: Integration of modeling, inference, planning
  - Posterior density structure determination and modeling
  - On-line inference and performance prediction
  - Optimized action selection
- Limitations:
  - Presence of sensor calibration errors
  - Complex noise and clutter limited environment
  - High measurement/scene dimensionality
- Components of research approach
  - Sequential resource allocation
    - Planning-optimized inference and Inference optimized planning
    - Sequential waveform design for ATE
  - Topological/structural modeling
    - Intrinsic dimensionality estimation
    - Structured dimensionality reduction
    - System feasibility analysis
  - Volumetric imaging and inverse scattering
    - Exploit target sparsity in imaging volume
    - Monotone sparsity-penalized iterative Born approximation
    - Scatterer confidence mapping
    - Action contingent performance prediction



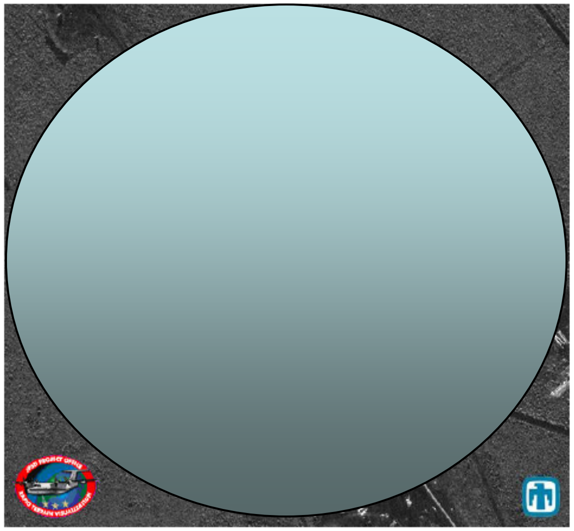
Agile Multi-Static Radar system illustration

This talk

Multistage adaptive SAR Image Acquisition

Images available at Sandia National Laboratories webs

Stage 1
Wide area search

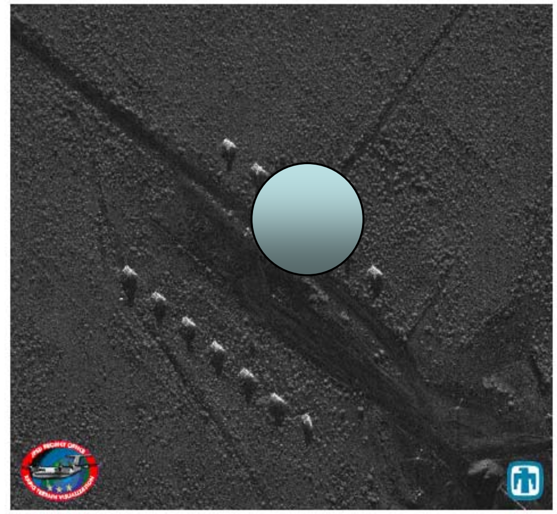


1[ft]=12[in] resolution

Stage 2
Refined search

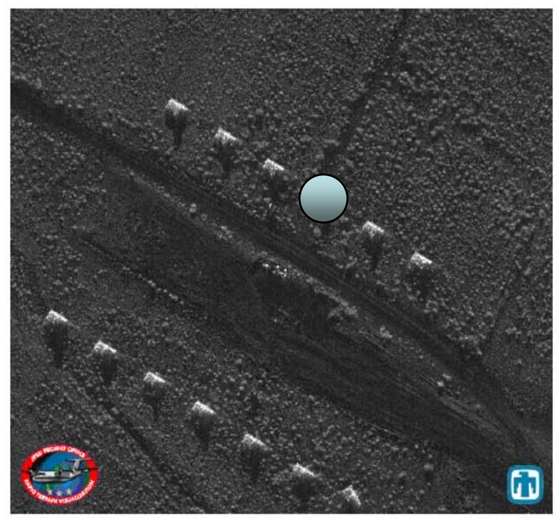
9[in] resolution

Stage 3
Refined search



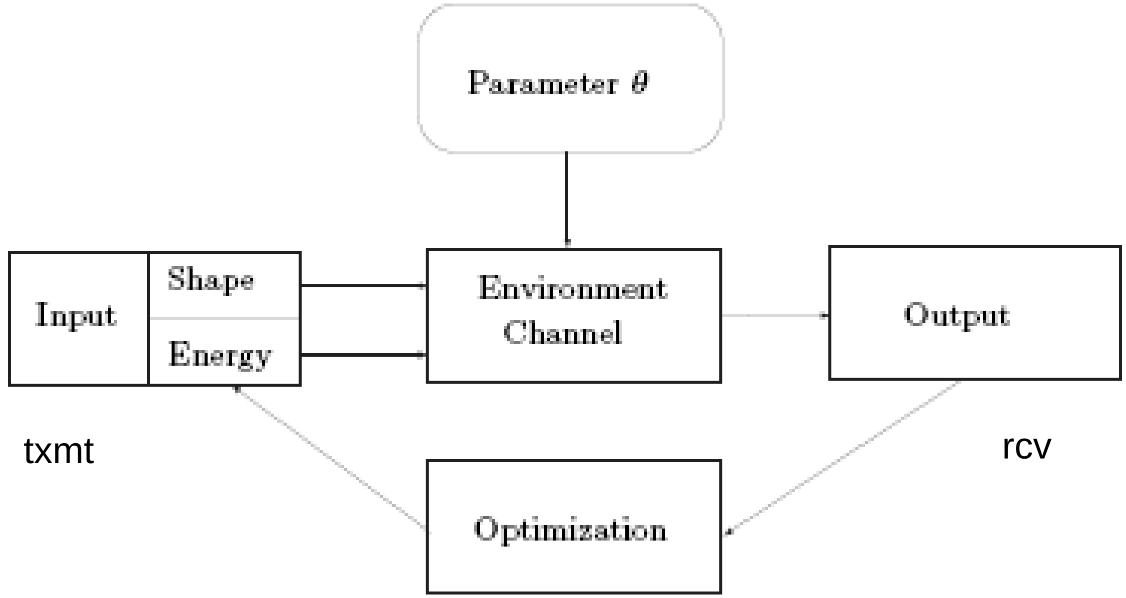
6[in] resolution

Stage 4
Refined search



4[in] resolution

II. Sequential Waveform Design

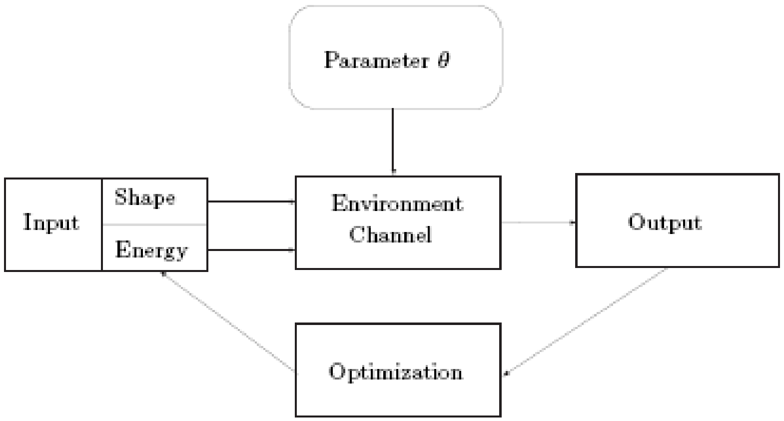


- Divide problem into two sub-problems:

  1. How to distribute energy over space (waveform shape design)?
  2. How to distribute energy over time (waveform amplitude design)?

- First we focus on 2

Sequential Waveform Design: Linear Model



Estimation in linear models

N-step measurement process:

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{H}(\mathbf{x}_1)\theta + \mathbf{n}_1 \\ \mathbf{y}_2 &= \mathbf{H}(\mathbf{x}_2(\mathbf{y}_1))\theta + \mathbf{n}_2 \\ &\vdots \\ \mathbf{y}_N &= \mathbf{H}(\mathbf{x}_N(\mathbf{y}_1, \ldots, \mathbf{y}_{N-1}))\theta + \mathbf{n}_N \end{aligned}$$

- Unknown parameter vector: $\boldsymbol{\theta} = [\theta_1, \theta_2 \ldots, \theta_M]^T$
- Average energy constraint: $\mathrm{E}\left[\sum_{i=1}^{N} \|\mathbf{x}_i(\mathbf{y}_1, \ldots, \mathbf{y}_{i-1})\|^2\right] \le E_0$
- Design $\mathbf{x}_1, \mathbf{x}_2(\mathbf{y}_1), \mathbf{x}_3(\mathbf{y}_1, \mathbf{y}_2), \ldots, \mathbf{x}_N(\mathbf{y}_1, \ldots, \mathbf{y}_{N-1})$ to maximize performance

- Compare to standard peak and average power constraints, e.g. Schweppe or Kershaw.

F. C. Schweppe and D. L. Gray, "Radar signal design subject to simultaneous peak and average constraints," *IEEE Trans. Inform. Theory*, vol. IT-12, pp. 13–26, 1966.

D. J. Kershaw and R. J. Evans, "Optimal waveform selection for tracking systems," *IEEE Trans. Inform. Theory*, vol. 40, no. 5, pp. 1536–1550, 1994.

* R. Rangarajan etal, IEEE Journ Select. Topics in SP, July 2007.

1. How to distribute energy over time?

• **Without feedback**, performance of the optimal estimator/detector only depends on time averaged transmitted energy E0

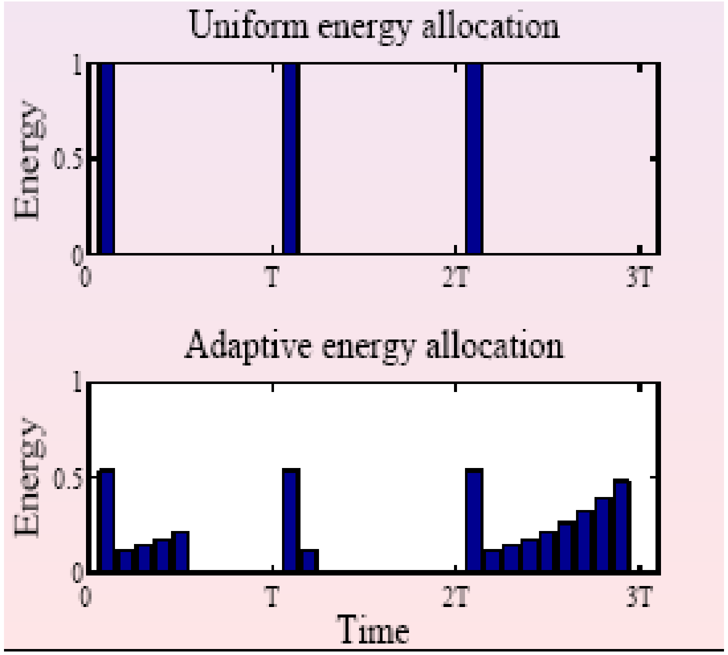
$$\text{MSE}=1/\text{SNR}, \text{ where SNR}=\frac{E_0}{\sigma^2}$$

any energy allocation strategy is as good as any other.

• **With feedback**, performance of the optimal estimator/detector depends on energy allocation over time

adaptive allocation strategy can provide enhanced performance

• **Q. Given N time slots for transmission, how to select sequence of transmitted rms amplitudes to maximize optimal estimator performance?**



Energy allocated over time to particular voxel

Optimal 2-step solution

- 2 step observation sequence

$$\mathbf{y}_1 = \mathbf{h}_1(\mathbf{x}_1)\theta_1 + \mathbf{n}_1$$
$$\mathbf{y}_2 = \mathbf{h}_1(\mathbf{x}_2(\mathbf{y}_1))\theta_1 + \mathbf{n}_2.$$

$$\mathbf{x}_1 = \sqrt{E_0}\alpha_1 \mathbf{v}_{\mathrm{m}}$$
$$\mathbf{x}_2(\mathbf{y}_1) = \sqrt{E_0}\alpha_2(\mathbf{y}_1)\mathbf{v}_{\mathrm{m}}$$

- MLE based on 2 step observations is

$$\hat{\theta}_1^{(2)} = \frac{\mathbf{h}_1(\mathbf{x}_1)^H \mathbf{y}_1 + \mathbf{h}_1(\mathbf{x}_2)^H \mathbf{y}_2}{||\mathbf{h}_1(\mathbf{x}_1)||^2 + ||\mathbf{h}_1(\mathbf{x}_2)||^2}$$

- MSE of MLE is

$$\mathrm{MSE}^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = \mathrm{E}\left[\frac{|\mathbf{h}_1(\mathbf{x}_1)^H \mathbf{n}_1 + \mathbf{h}_1(\mathbf{x}_2)^H \mathbf{n}_2|^2}{(||\mathbf{h}_1(\mathbf{x}_1)||^2 + ||\mathbf{h}_1(\mathbf{x}_2)||^2)^2}\right]$$

- Objective: find amplitudes $\alpha_1, \alpha_2(y_1)$ that minimize MSE subject to constraint $\mathrm{E}\left[\alpha_1^2 + \alpha_2^2(\mathbf{y}_1)\right] \leq 1.$

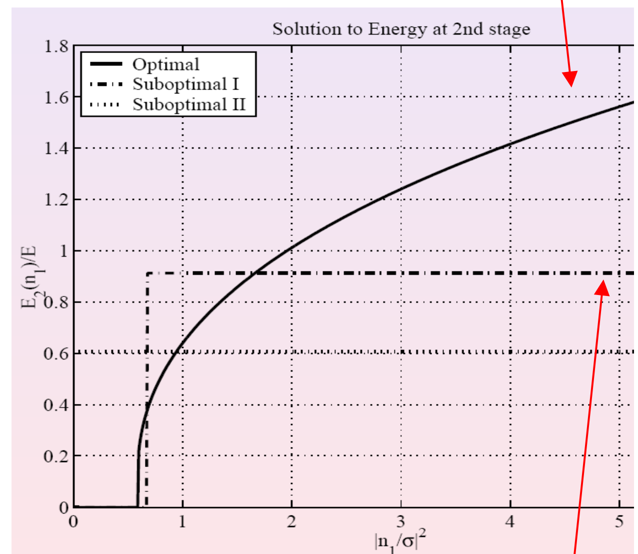
$$\mathrm{E}\left[\frac{|\mathbf{h}_1(\mathbf{x}_1)^H \mathbf{n}_1 + \mathbf{h}_1(\mathbf{x}_2)^H \mathbf{n}_2|^2}{(||\mathbf{h}_1(\mathbf{x}_1)||^2 + ||\mathbf{h}_1(\mathbf{x}_2)||^2)^2}\right] + \gamma\left(\alpha_1^2 + \mathrm{E}\left[\alpha_2^2(\mathbf{y}_1)\right]\right)$$

Omniscient 2-step Strategy

- If parameter θ is known and $\mathbf{h}$ is linear, then optimal strategy is soft thresholding

$$\alpha_2^* = \alpha_1^* \sqrt{g\left(\frac{\mathbf{h}_1(\mathbf{v}_m)^H \mathbf{n}_1}{\|\mathbf{h}_1(\mathbf{v}_m)\|\sigma} - 1\right)}$$

Optimal soft threshold



Subptimal hard threshold

- Where g is solution to

$$g^3 - \frac{1}{\gamma'} g + 2\frac{1 - |\tilde{n}_1|^2}{\gamma'} = 0$$

$$\gamma' = \gamma \alpha_1^2 \|\mathbf{h}(\mathbf{v}_m)\|^2 / \sigma^2 \qquad \tilde{\mathbf{n}}_1 = \frac{\mathbf{h}_1(\mathbf{v}_m)^H \mathbf{n}_1}{\|\mathbf{h}_1(\mathbf{v}_m)\|\sigma}$$

- Omniscient 2-step performance

- Design $E_1, E_2(\mathbf{y}_1)$ optimally.
- $E_1 \approx 0.55\ E_0$.
- $\mathrm{E}\left[E_2(\mathbf{y}_1)\right] \approx 0.45\ E_0$.
- $MSE_2 \approx 0.68/SNR = 0.68\ MSE_1$.
- About 1.6dB gain in performance.

From 2 steps to Nx2 steps

- Perform N independent 2-step experiments each allocated energy E_0/N
- At each step form ML estimate $\widehat{\theta}^{(k)}$ of parameter θ
- When N-fold experiment terminates, form global estimate

$$\widehat{\theta}^{(2N)} = \frac{1}{2N}\sum_{k=1}^{2N}\widehat{\theta}^{(k)}$$

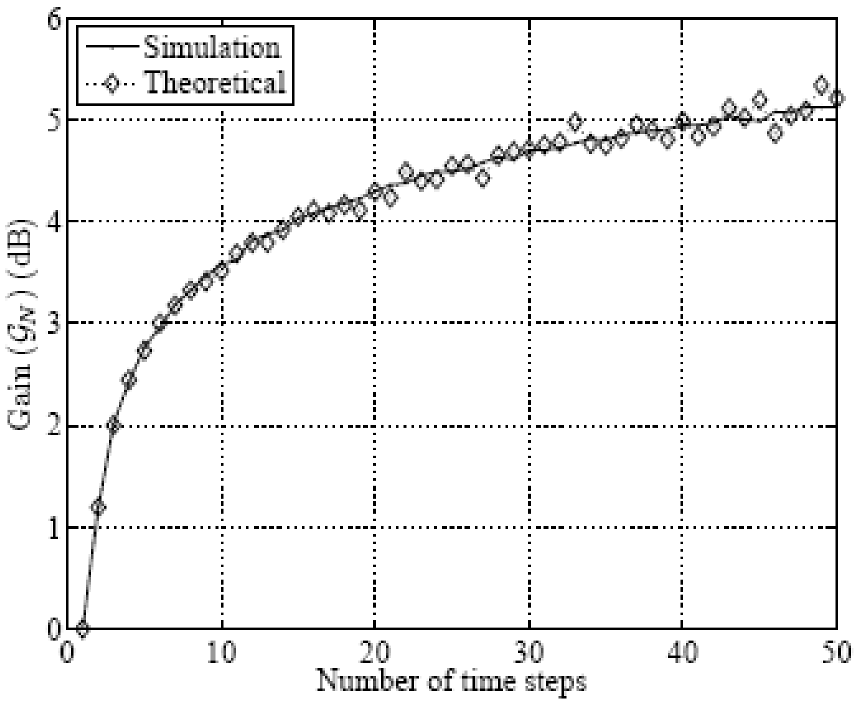
- The MSE of the global estimate will satisfy (here z denotes θ)

$$\mathrm{MSE}^{(2N)}(z) \times \mathrm{SNR}^{(2N)}(z) = \mathrm{MSE}^{(2)}(z/\sqrt{N})\mathrm{SNR}^{(2)}(z/\sqrt{N})$$

- And as N goes to infinity the minimal MSE is acheived

$$\mathrm{MSE}^{(2N)}(z) \times \mathrm{SNR}^{(2N)}(z) \to \eta^*$$

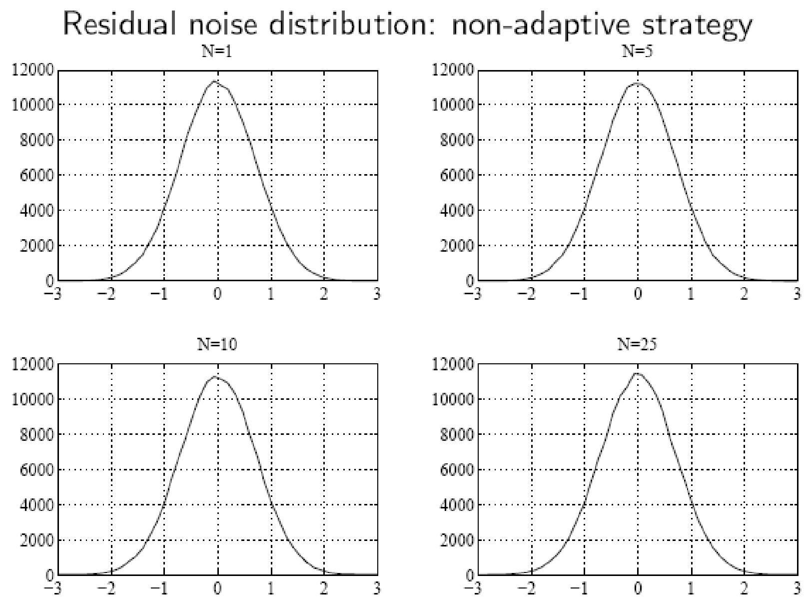
Nx2 Step Adaptive Gain

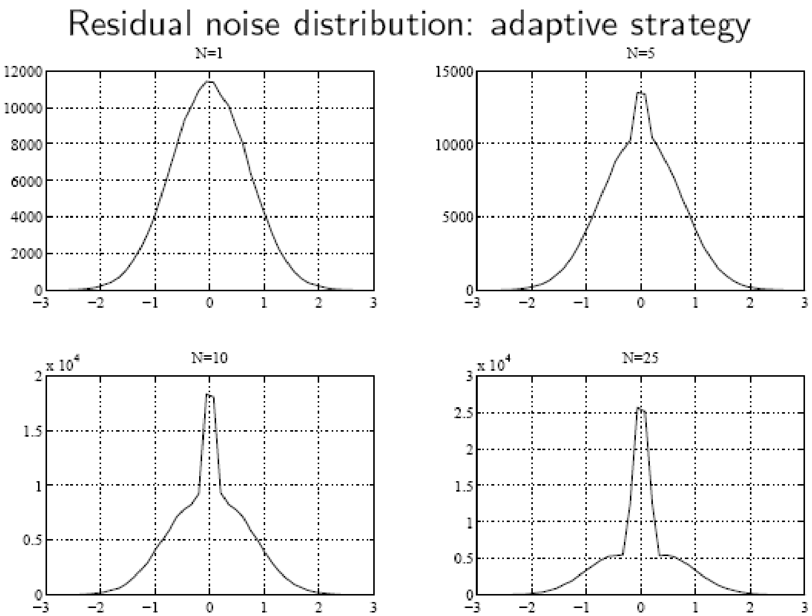


Highlights

- Optimal two-step strategy: 1.67dB gain
- Suboptimal N-strategy: over 5dB gain
- Closed-form solutions to design parameters
  - Easily implementable
- Closed-form expressions for error and achievable performance

Nx2 Step Residual Error Distribution



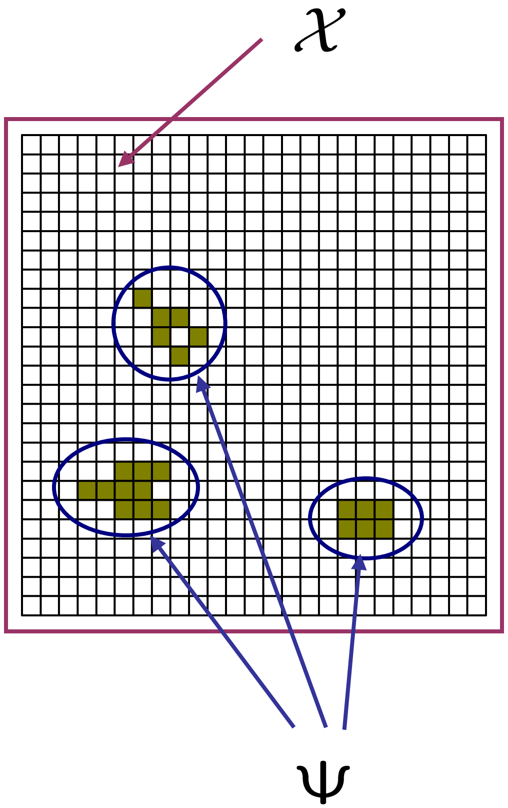


Gaussian distribution has maximum entropy for fixed variance

Discussion

- **Take-home message**: adaptively distributing transmit energy over time achieves MSE performance gains of 5dB
- Equivalently, for given level of MSE can reduce acquisition time by factor of 3
- Hard thresholding approximation to optimal soft thresholding function entails only minor loss in performance.
- Computational complexity is low
- Implementation requires knowledge of noise variance and **h** (Greens function).
- Extensions (Rangarajan:Thesis07)
  - Non-linear models: Rayleigh fading environments
  - Vector valued parameters: region estimation, extended objects
  - Other ATE objectives: detection, classification
- Similar trends in performance gain are shown in the above cases

2. How to distribute energy over space?



- Set of all cells $\mathcal{X} = \{1, 2, \ldots, Q\}$
- Unknown ROI $\Psi \subseteq \mathcal{X}$ $\quad I_i = I(i \in \Psi), \; i \in \mathcal{X}.$
- Image sparsity factor $p = 1 - |\Psi|/|\mathcal{X}|$
- Spatio-temporal energy allocation policy

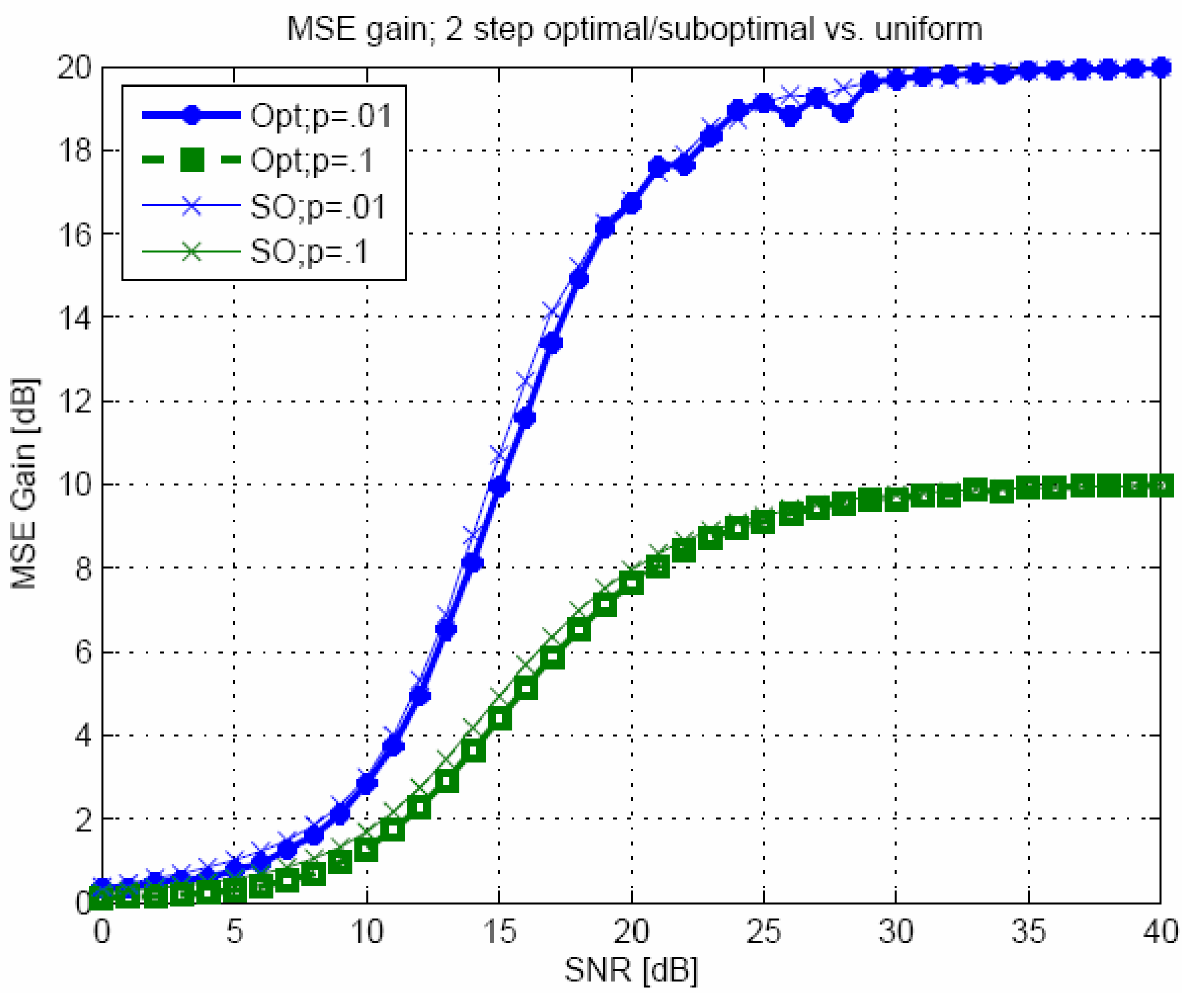
$$E_t(i) = E(t, i, y(1), \ldots, y(t-1))$$

$$E_t(i) \geq 0, \;\; \sum_t E_t(i) = E_i, \;\; \sum_i E_i = E_T.$$

- Observations $\sim p(\{y(t)\}|\{I_i\}, \{E_t(i)\})$
- Uniform spatial allocation: $E_i = E_T/|\mathcal{X}|.$
- Ideal spatial allocation: $E_i = E_T/|\Psi| \; I_i.$
- Optimal N-step allocation: multistage stochastic control problem
- Simpler objective: find two-step optimal allocation that minimizes (weighted avg terminal estimator MSE)

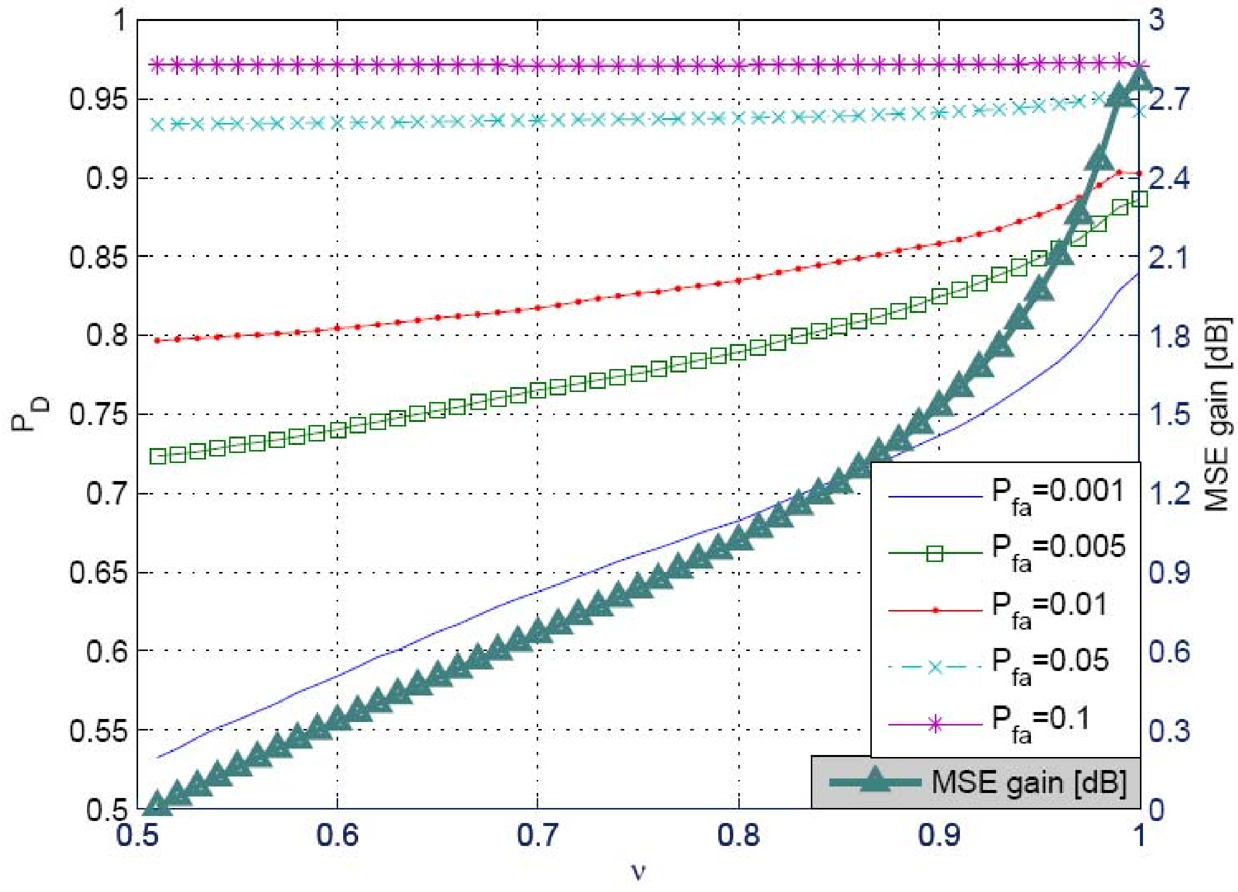
$$J = \mathbf{E}\Big[\sum_i \frac{\nu I_i + (1-\nu)(1-I_i)}{E_i}\Big], \;\; s.t. \;\; \sum_i E_i = E_T$$

MSE Gain (ν=1, Q=8192)



For known target, performance best for ν=1

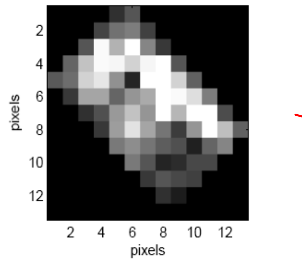
$$\nu = \begin{cases} < 1, & \textit{induce exploration} \\ 1, & \textit{exploitation only} \end{cases}$$

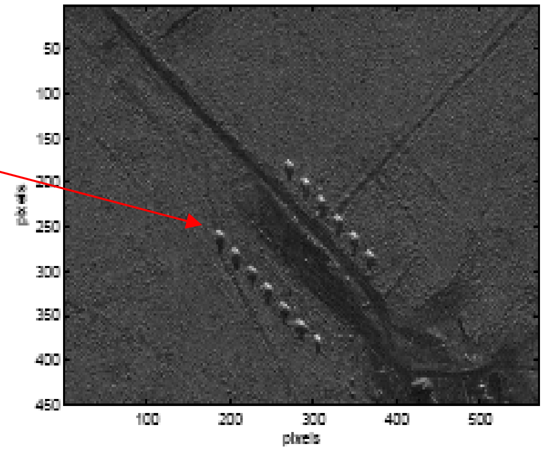


Discussion

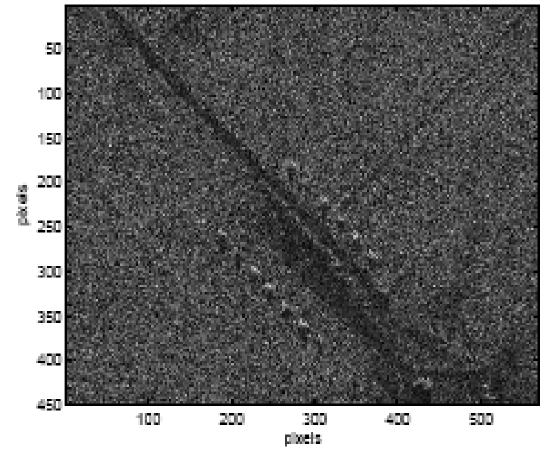
- **Take home message**: with sparse images, adaptive MSE gain is order 1/p
- Objective function J only depends on the cumulative energy allocated to each voxel in the image volume (deferred reward)
- Features of optimal two-step policy
  - Waterfilling policy: allocates energy to regions in proportion to posterior probability of containing targets
  - minimizes the Chernoff bound and the CRB under suitable Gaussian measurement model.
  - can be interpreted as Bayesian version of Posner's two–step likelihood-based search algorithm: E. Posner, "Optimal search procedures," *IEEE Transactions on Information Theory*, vol. 9, no. 3, pp. 157–160, July 1963.
- Computational requirements of two-step optimal policy
  - Computation of posterior distribution of ROI at a given voxel (O(Q))
  - Computation of threshold k0 (O(logQ))
  - Specification of parameters p, ν
- Parameter ν controls the tradeoff between "exploration and exploitation" in searching over the image volume

SAR Imaging Example

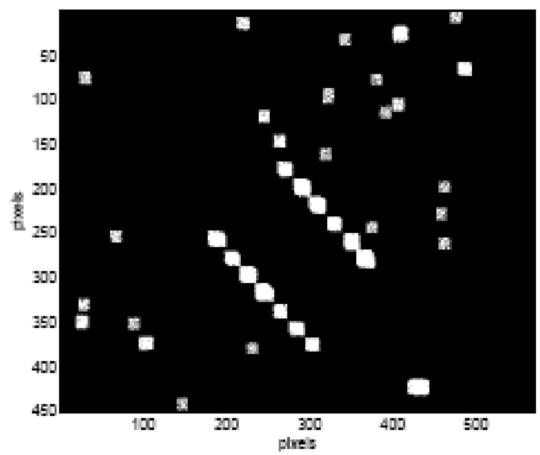




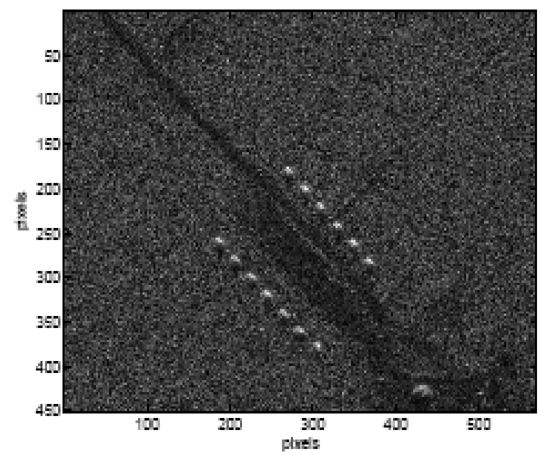
Original



Wide area acquisition

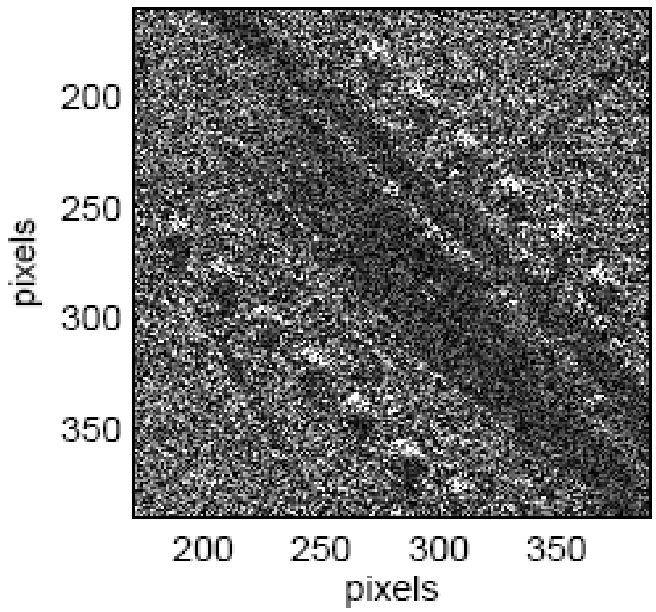


Energy allocation at 2nd step

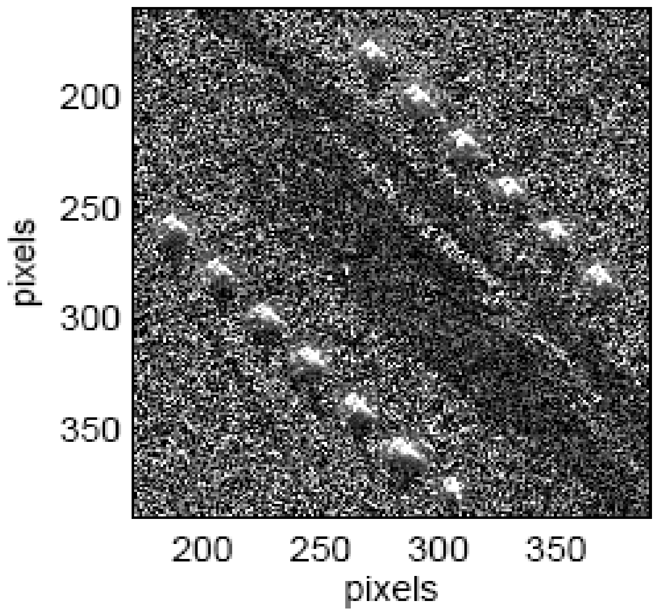


Optimal 2-step acquisition

Comparisons



(a)



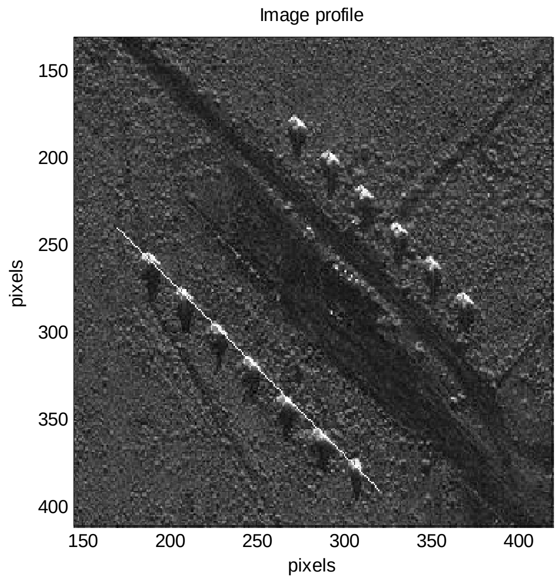
(b)

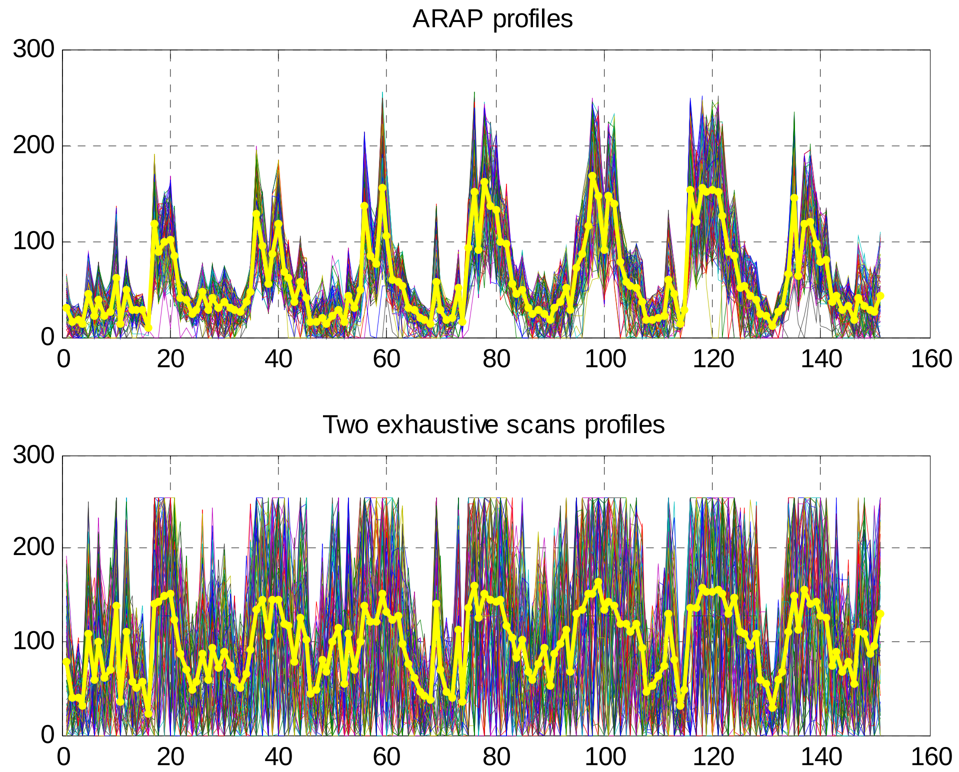
Wide area SAR acquisition

Optimal two step SAR Acquisition (ARAP)

Overall energy allocated is identical in both cases

Profile Comparisons





Punch Lines

- Optimal dwell (temporal energy allocation) [Rangarajan07]
  - Sequential decisions framework
  - Packetization of energy with optimal stopping can provide gains of more than 5dB in MSE, 2dB in detection performance
- Optimal search (spatial energy allocation) [Bashan07]
  - Bayesian adaptive sampling framework
  - Near-optimal energy/time allocation achievable by linear complexity algorithm

Related Activities

- Inverse scattering with statistical priors (ARO MURI - Harmon)
  - Optimization transfer approach combines iterative Born and sparsity inducing prior into monotone single iteration [Raich, Bagci, Michielssen].
- Blind molecular imaging (ARO MURI - Prater)
  - Bayesian likelihood model with LAZE sparsity-inducing prior and partially known imaging PSF [Raich, Herrity].
- Fundamental limits of MIMO radar imaging (GD)
  - Hybrid CR bound derived for MIMO radar [M. Davis]
- Topological inference and information theory
  - Entropic graphs for classification [S.J. Hwang, S. Damelin]
- Graphical models for manifold+structure learning
  - Models embedded in lower dimensions [V. Chandrasekaran]
- Information theoretic limits on networked ATE
  - Rate distortion theory for belief propagation [J. Fisher]

Foundations and Applications of Sensor Management

Publisher: Springer - To appear late 2007

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Stan Musick - AFRL
Yan Zhang - Humana

Publications (2006-2007)

- Appeared
  - Optimal Sequential Energy Allocation for Inverse Problems Rangarajan, R.; Raich, R.; Hero, A.O.; Selected Topics in Signal Processing, IEEE Journal of Volume 1, Issue 1, June 2007

    Page(s):67 - 78.
  - R. Rangarajan, "*Resource constrained adaptive sensing* ," PhD Thesis, University of Michigan, July 2007.
- In cogitation or preparation
  - E. Bashan, R. Raich and A.O. Hero, "Efficient search under resource constraints," working draft.
  - A.O. Hero, V. Chandrasekaran, and A. Willsky, "Learning embedded graphical models," working draft.
  - R. Raich, J. Costa, S. Damelin, and A.O. Hero, "Classification constrained dimensionality reduction," working draft.