Adaptive radar sensing strategies

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AFOSR MURI

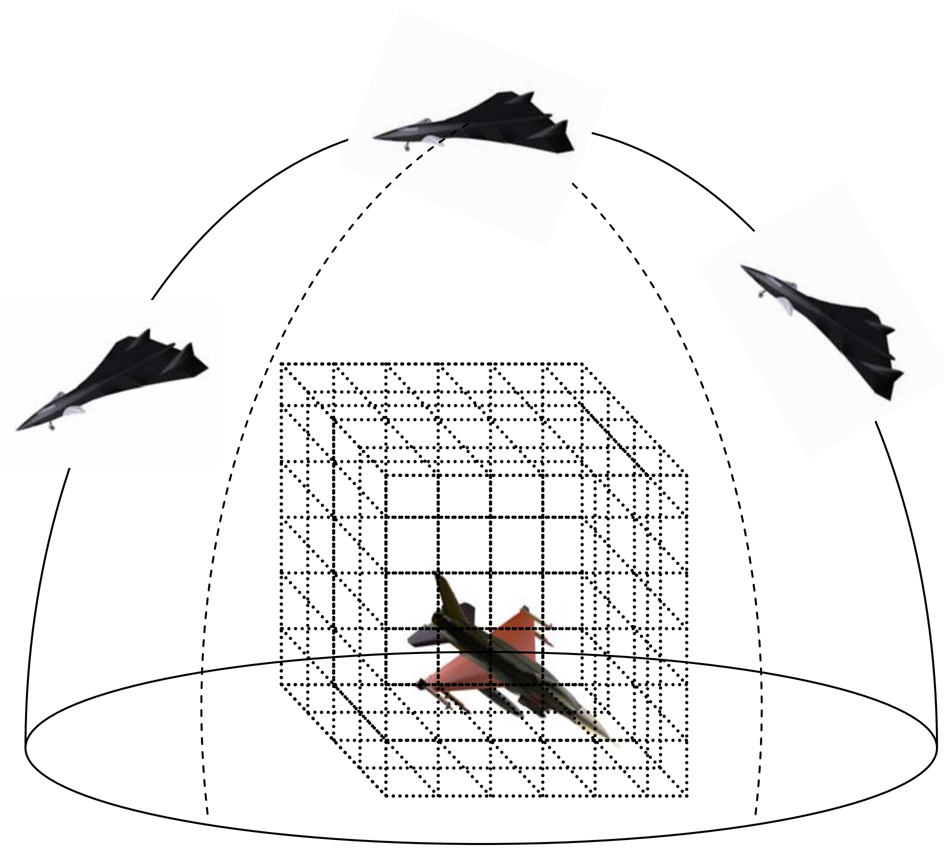
Integrated fusion, performance prediction,
and sensor management for ATE
(PI: R. Moses)

Outline

- ATE Vision and Research Approach
- Sequential Resource Allocation
  - Sequential Waveform Design for ATE
- Structured Dimensionality Reduction (preliminary)
- Information items
  - Synergistic Activities
  - SM book to appear
  - Publications

I. ATE Vision and Research Approach

- ATE: Integration of modeling, inference, planning
  - Posterior density structure determination and modeling
  - On-line inference and performance prediction
  - Optimized action selection
- Limitations:
  - Presence of sensor calibration errors
  - Complex noise and clutter limited environment
  - High measurement/scene dimensionality
- Components of research approach
  - Sequential resource allocation
    - Planning-optimized inference and Inference optimized planning
    - Sequential waveform design for ATE
  - Topological/structural modeling
    - Intrinsic dimensionality estimation
    - Structured dimensionality reduction
    - System feasibility analysis
  - Volumetric imaging and inverse scattering
    - Exploit target sparsity in imaging volume
    - Monotone sparsity-penalized iterative Born approximation
    - Scatterer confidence mapping
    - Action contingent performance prediction



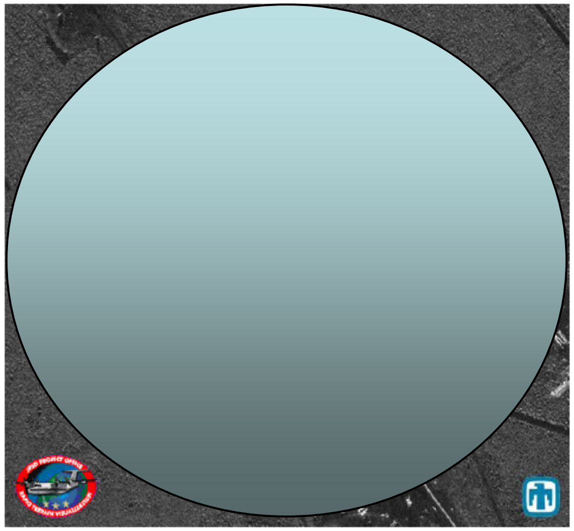
Agile Multi-Static Radar system illustration

This talk

Multistage adaptive SAR Image Acquisition

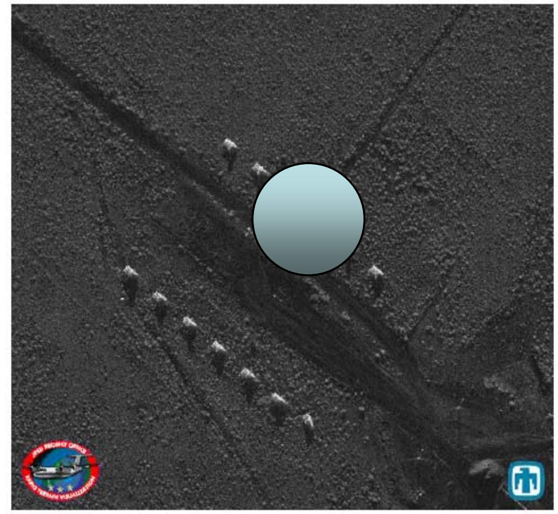
Images available at Sandia National Laboratories webs

Stage 1
Wide area search



1[ft]=12[in] resolution

Stage 2
Refined search

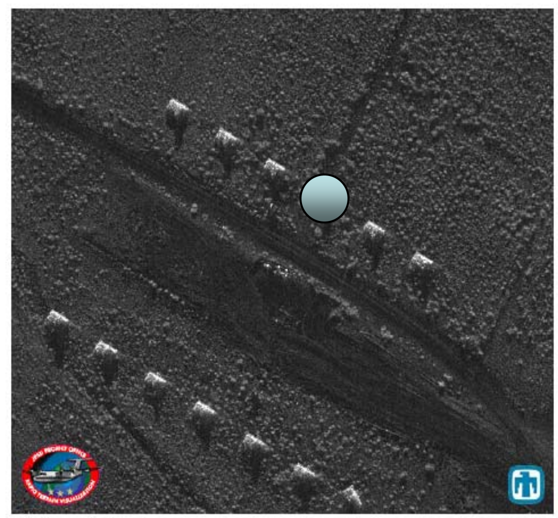


6[in] resolution

Stage 3
Refined search

9[in] resolution

Stage 4
Refined search



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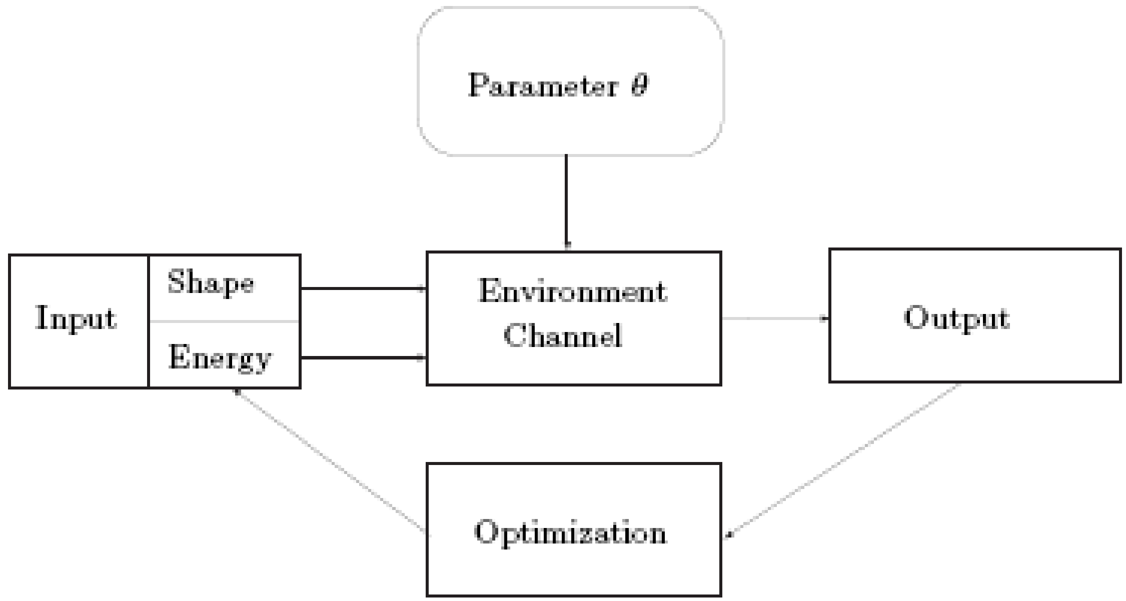
II. Sequential Waveform Design

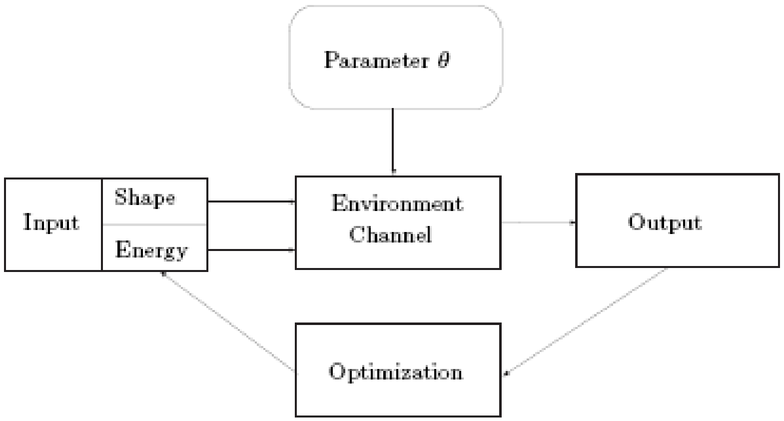


- Divide problem into two sub-problems:

  1. How to distribute energy over space (waveform shape design)?
  2. How to distribute energy over time (waveform amplitude design)?

- First we focus on 2

Sequential Waveform Design: Scalar Linear Model



Estimation in linear models

N-step measurement process:

$$
\begin{aligned}
\mathbf{y}_1 &= \mathbf{H}(\mathbf{x}_1)\theta + \mathbf{n}_1 \\
\mathbf{y}_2 &= \mathbf{H}(\mathbf{x}_2(\mathbf{y}_1))\theta + \mathbf{n}_2 \\
&\vdots \\
\mathbf{y}_N &= \mathbf{H}(\mathbf{x}_N(\mathbf{y}_1, \ldots, \mathbf{y}_{N-1}))\theta + \mathbf{n}_N
\end{aligned}
$$

- Unknown parameter vector: $\boldsymbol{\theta} = [\theta_1, \theta_2 \ldots, \theta_M]^T$
- Average energy constraint: $\mathrm{E}\left[\sum_{i=1}^{N} \|\mathbf{x}_i(\mathbf{y}_1, \ldots, \mathbf{y}_{i-1})\|^2\right] \leq E_0$
- Design $\mathbf{x}_1, \mathbf{x}_2(\mathbf{y}_1), \mathbf{x}_3(\mathbf{y}_1, \mathbf{y}_2), \ldots, \mathbf{x}_N(\mathbf{y}_1, \ldots, \mathbf{y}_{N-1})$ to maximize performance

• Compare to standard peak and average power constraints, e.g. Schweppe or Kershaw.

F. C. Schweppe and D. L. Gray, "Radar signal design subject to simultaneous peak and average constraints," *IEEE Trans. Inform. Theory*, vol. IT-12, pp. 13–26, 1966.

D. J. Kershaw and R. J. Evans, "Optimal waveform selection for tracking systems," *IEEE Trans. Inform. Theory*, vol. 40, no. 5, pp. 1536–1550, 1994.

* R. Rangarajan etal, IEEE Journ Select. Topics in SP, July 2007.

1. How to distribute energy over time?

• **Without feedback**, performance of the optimal estimator/detector only depends on time averaged transmitted energy E0

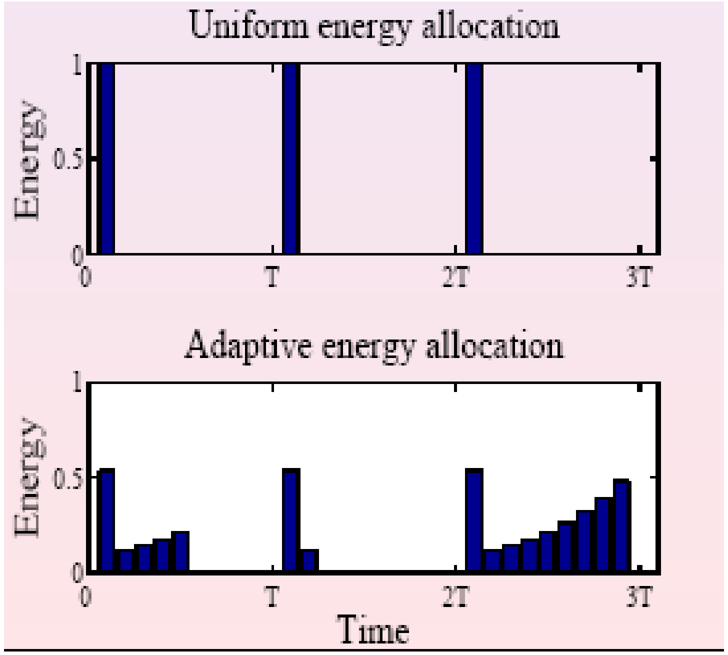
$$\text{MSE}=1/\text{SNR}, \text{ where } \text{SNR}=\frac{E_0}{\sigma^2}$$

any energy allocation strategy is as good as any other.

• **With feedback**, performance of the optimal estimator/detector depends on energy allocation over time

adaptive allocation strategy can provide enhanced performance

• Q. Given N time slots for transmission, how to select sequence of transmitted rms amplitudes to maximize optimal estimator performance?



Energy allocated over time to particular voxel

Optimal 2-step solution

- 2 step observation sequence

$$\mathbf{y}_1 = \mathbf{h}_1(\mathbf{x}_1)\theta_1 + \mathbf{n}_1$$
$$\mathbf{y}_2 = \mathbf{h}_1(\mathbf{x}_2(\mathbf{y}_1))\theta_1 + \mathbf{n}_2.$$

$$\mathbf{x}_1 = \sqrt{E_0}\alpha_1\mathbf{v}_{\mathrm{m}}$$
$$\mathbf{x}_2(\mathbf{y}_1) = \sqrt{E_0}\alpha_2(\mathbf{y}_1)\mathbf{v}_{\mathrm{m}}$$

- MLE based on 2 step observations is

$$\hat{\theta}_1^{(2)} = \frac{\mathbf{h}_1(\mathbf{x}_1)^H\mathbf{y}_1 + \mathbf{h}_1(\mathbf{x}_2)^H\mathbf{y}_2}{||\mathbf{h}_1(\mathbf{x}_1)||^2 + ||\mathbf{h}_1(\mathbf{x}_2)||^2}$$

- MSE of MLE is

$$\mathrm{MSE}^{(2)}(\mathbf{x}_1, \mathbf{x}_2) = \mathrm{E}\left[\frac{|\mathbf{h}_1(\mathbf{x}_1)^H\mathbf{n}_1 + \mathbf{h}_1(\mathbf{x}_2)^H\mathbf{n}_2|^2}{(||\mathbf{h}_1(\mathbf{x}_1)||^2 + ||\mathbf{h}_1(\mathbf{x}_2)||^2)^2}\right]$$

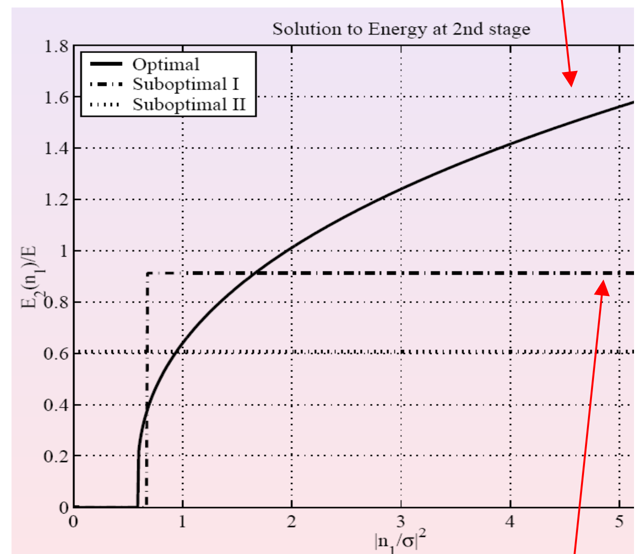
- Objective: find amplitudes $\alpha_1, \alpha_2(y_1)$ that minimize MSE subject to constraint $\mathrm{E}\left[\alpha_1^2 + \alpha_2^2(\mathbf{y}_1)\right] \leq 1.$

$$\mathrm{E}\left[\frac{|\mathbf{h}_1(\mathbf{x}_1)^H\mathbf{n}_1 + \mathbf{h}_1(\mathbf{x}_2)^H\mathbf{n}_2|^2}{(||\mathbf{h}_1(\mathbf{x}_1)||^2 + ||\mathbf{h}_1(\mathbf{x}_2)||^2)^2}\right] + \gamma\left(\alpha_1^2 + \mathrm{E}\left[\alpha_2^2(\mathbf{y}_1)\right]\right)$$

Omniscient 2-step Strategy

- If parameter θ is known and **h** is linear, then optimal strategy is soft thresholding

$$\alpha_2^* = \alpha_1^* \sqrt{g\left(\frac{\mathbf{h}_1(\mathbf{v}_m)^H \mathbf{n}_1}{\mathbf{h}_1(\|\mathbf{v}_m)\|\sigma} - 1\right)}$$



- Where g is solution to

$$g^3 - \frac{1}{\gamma'} g + 2\frac{1 - |\tilde{n}_1|^2}{\gamma'} = 0$$

$$\gamma' = \gamma \alpha_1^2 \|\mathbf{h}(\mathbf{v}_m)\|^2/\sigma^2 \qquad \tilde{\mathbf{n}}_1 = \frac{\mathbf{h}_1(\mathbf{v}_m)^H \mathbf{n}_1}{\mathbf{h}_1(\|\mathbf{v}_m)\|\sigma}$$

- Omniscient 2-step performance

- Design $E_1, E_2(\mathbf{y}_1)$ optimally.
- $E_1 \approx 0.55\, E_0$.
- $\mathrm{E}\,[E_2(\mathbf{y}_1)] \approx 0.45\, E_0$.
- $MSE_2 \approx 0.68/SNR = 0.68\, MSE_1$.
- About 1.6dB gain in performance.

From 2 steps to Nx2 steps

- Perform N independent 2-step experiments allocating energy E_0/N to each of the N experiments.
- At each step form ML estimate $\widehat{\theta}^{(k)}$ of parameter θ
- When N-fold experiment terminates, form global estimate

$$\widehat{\theta}^{(2N)} = \frac{1}{2N} \sum_{k=1}^{2N} \widehat{\theta}^{(k)}$$

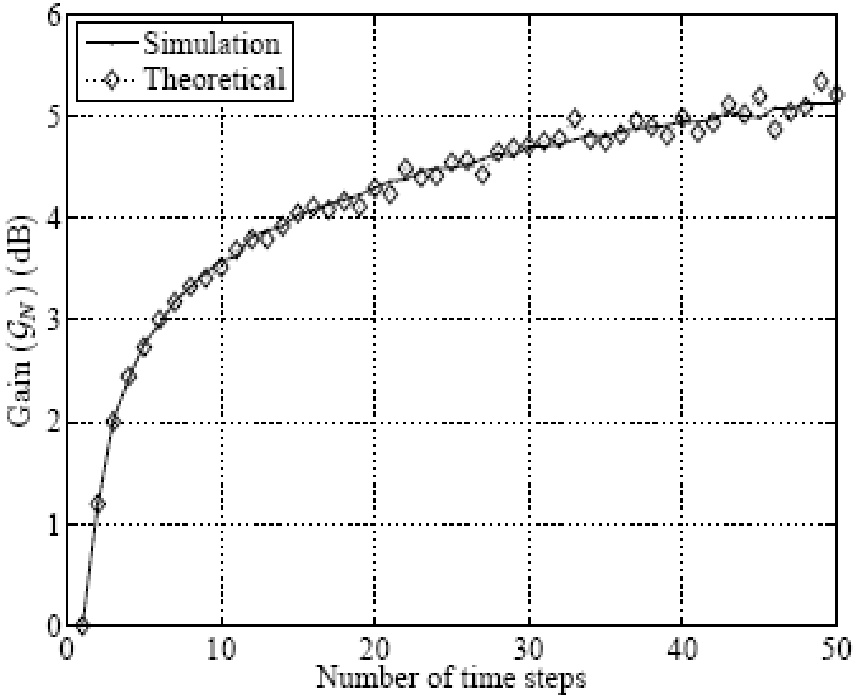
- The MSE of the global estimate will satisfy (here z denotes θ)

$$\mathrm{MSE}^{(2N)}(z) \times \mathrm{SNR}^{(2N)}(z) = \mathrm{MSE}^{(2)}(z/\sqrt{N})\mathrm{SNR}^{(2)}(z/\sqrt{N})$$

- And as N goes to infinity the minimal MSE is acheived

$$\mathrm{MSE}^{(2N)}(z) \times \mathrm{SNR}^{(2N)}(z) \to \eta^*$$

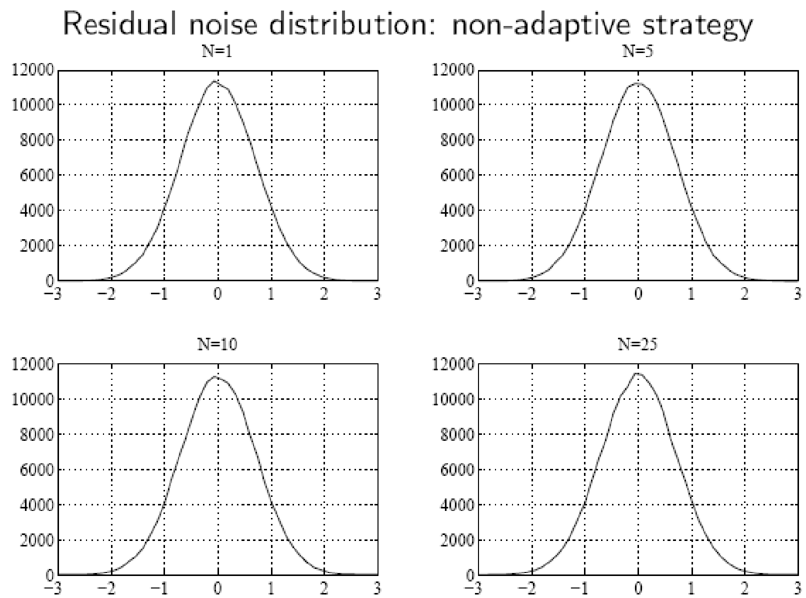
Nx2 Step Adaptive Gain



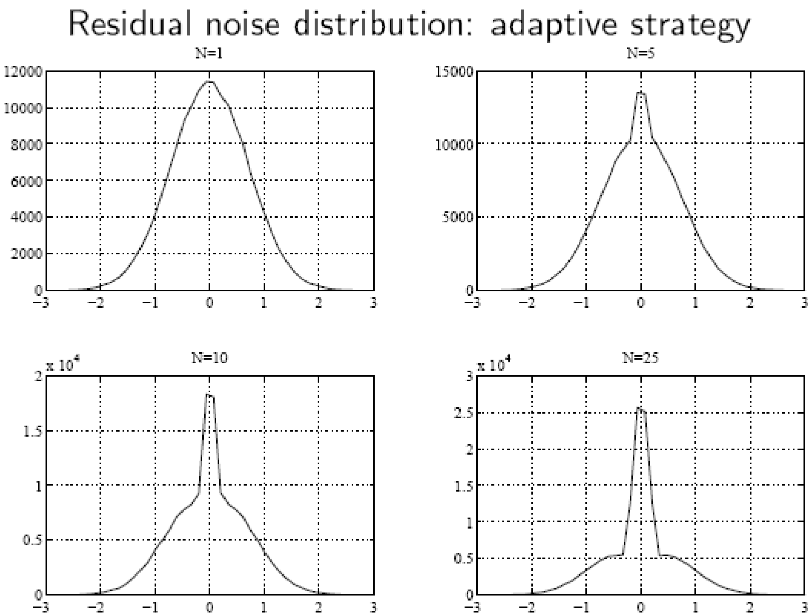
Highlights

- Optimal two-step strategy: 1.67dB gain
- Suboptimal *N*-strategy: over 5dB gain
- Closed-form solutions to design parameters
  - Easily implementable
- Closed-form expressions for error and achievable performance

Nx2 Step Residual Error Distribution



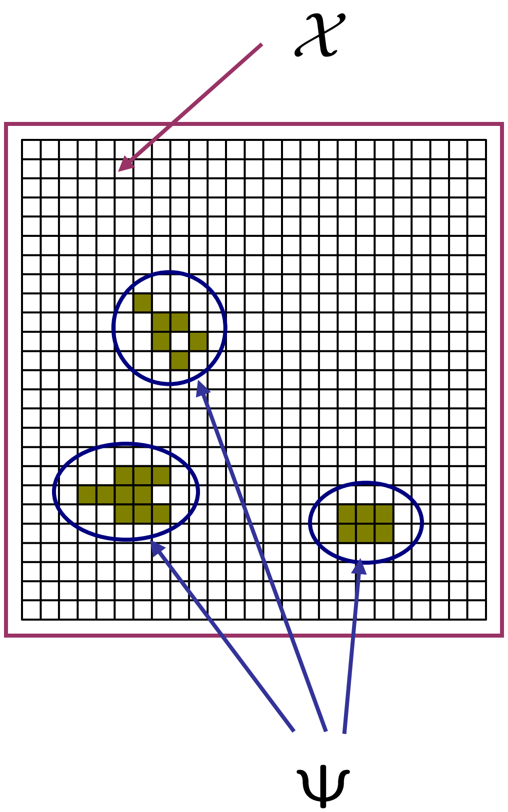
Gaussian distribution has maximum entropy for fixed variance



Discussion

- Take-home message: **we predict that adaptively distributing transmit energy over time achieves significant performance gains**
- Equivalently, for given level of system performance can reduce acquisition time
- Suboptimal hard thresholding approximation to optimal soft thresholding entails only minor loss in performance.
- Computational complexity is linear in number of steps N

2. How to distribute energy over space?



- Set of all cells $\mathcal{X} = \{1, 2, \ldots, Q\}$
- ROI $\Psi \subseteq \mathcal{X}$
- ROI indicator $I_i = I(i \in \Psi), \; i \in \mathcal{X}.$
- Spatio-temporal energy allocation policy

$$E_t(i) = E(t, i, y(1), \ldots, y(t-1))$$

$$E_t(i) \geq 0, \;\; \sum_t E_t(i) = E_i, \;\; \sum_i E_i = E_T.$$

- Observations $\sim p(\{y(t)\}|\{I_i\}, \{\lambda(i,t)\})$
- Uniform spatial allocation: $E_i = E_T/|\mathcal{X}|.$
- Ideal spatial allocation: $E_i = E_T/|\Psi| \; I_i.$
- Optimal N-step allocation: multistage stochastic control problem
- Simpler objective: find two-step optimal allocation that minimizes

$$J = \mathbf{E}\left[\sum_i \frac{\nu I_i + (1-\nu)(1-I_i)}{E_i}\right], \;\; s.t. \;\; \sum_i E_i = E_T$$

Optimal allocation strategy

Step 1: Allocate ε_1^* to each cell.

Step 2: Given $\boldsymbol{y}(1)$ derive w_i via $P_{I_i|y_i(1)}$ defined in (24), then sort the w_i's.

Step 3: Use ε_1^* and the ordered statistic $w_{\tau(i)}$ to derive k_0 using (27) and (28).

Step 4: Given k_0, define $\varepsilon(i,2)$ the allocated energy to cell i as

$$\varepsilon(\tau(i),2) = \left(\frac{\varepsilon_T - k_0\varepsilon_1^*}{\sum_{j=k_0+1}^{Q}\sqrt{w_{\tau(j)}}}\sqrt{w_{\tau(i)}} - \varepsilon_1^*\right) I(i > k_0). \qquad (32)$$

$$w_i = \nu p_{I_i|y(1)} + (1-\nu)(1 - p_{I_i|y(1)}).$$

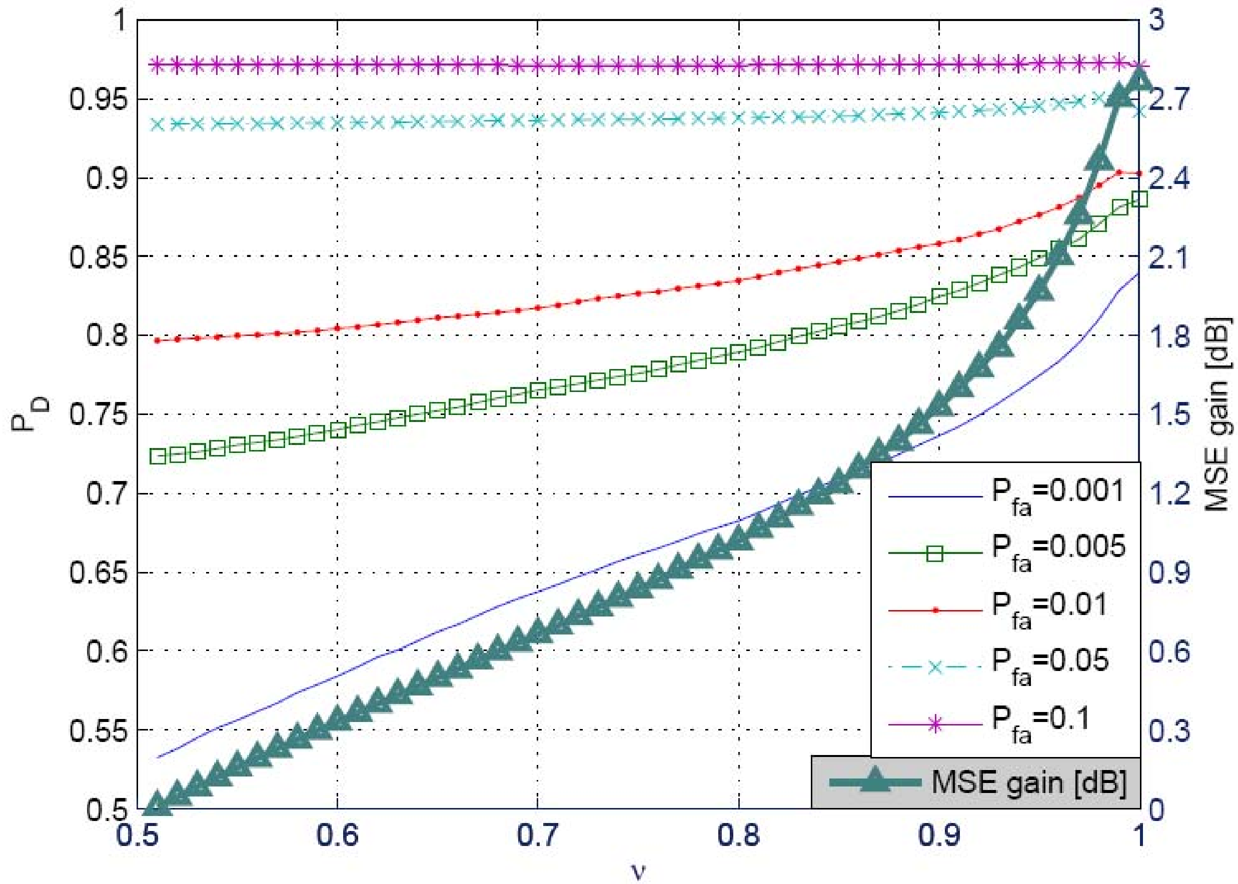
$$\frac{\sum_{i=k_0+1}^{Q}\sqrt{w_{\tau(i)}}}{\sqrt{w_{\tau(k_0+1)}}} < \frac{\varepsilon_T}{\varepsilon_1} - k_0 \leqslant \frac{\sum_{i=k_0+1}^{Q}\sqrt{w_{\tau(i)}}}{\sqrt{w_{\tau(k_0)}}}. \qquad (28)$$

Discussion

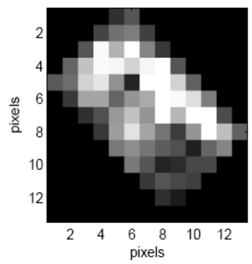
- Objective function J only depends on the cumulative energy allocated to each voxel in the image volume (deferred reward)
- Features of optimal two-step policy
  - assigns energy to regions with high posterior probability of containing targets
  - minimizes the Chernoff bound and the CRB under suitable Gaussian measurement model.
  - an index policy with threshold k0
  - can be interpreted as Bayesian version of Posner's two–step likelihood-based search algorithm: E. Posner, "Optimal search procedures," *IEEE Transactions on Information Theory*, vol. 9, no. 3, pp. 157–160, July 1963.
- Computational requirements of two-step optimal policy
  - Computation of posterior distribution of scatter centers at a given voxel (O(N))
  - Computation of threshold k0 (O(NlogN))
  - Specification of parameter ν
- Parameter ν controls the tradeoff between "exploration and exploitation" in searching over the image volume

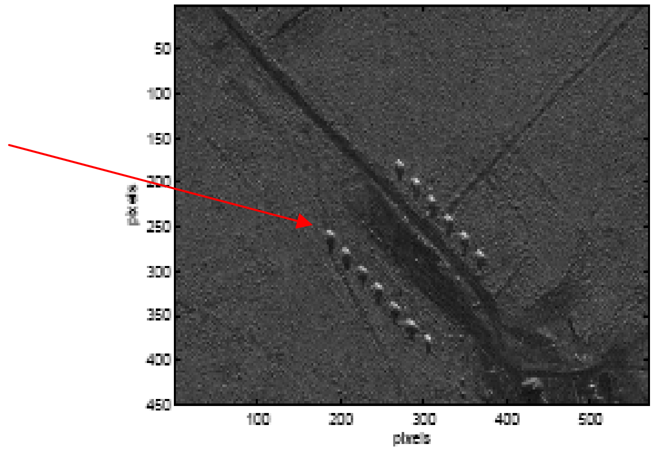
For known target task performance optimized for $\nu=1$

$$\nu = \begin{cases} < 1, & induce\ exploration \\ 1, & exploitation\ only \end{cases}$$

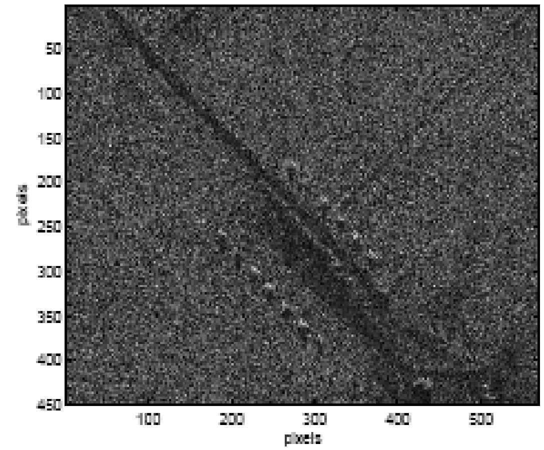


SAR Imaging Example

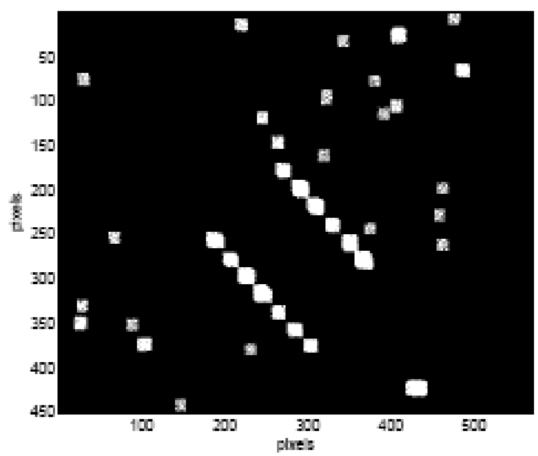




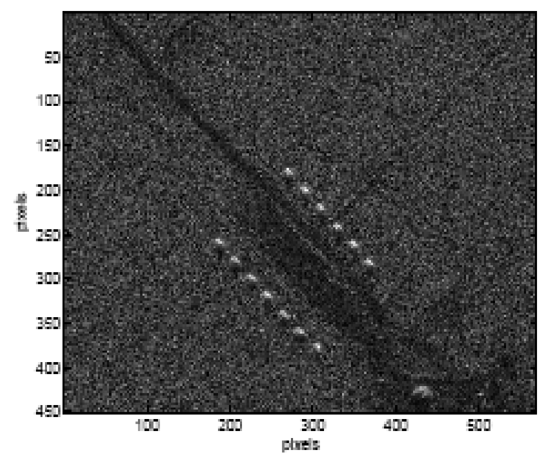
Original



Wide area acquisition

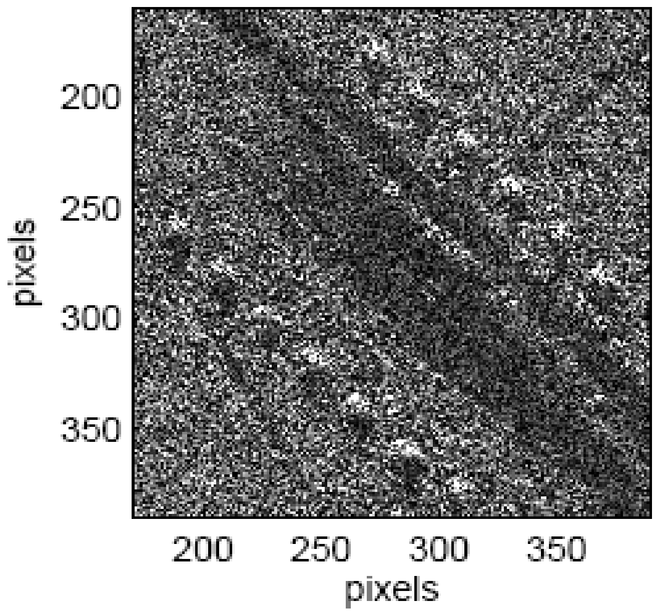


Energy allocation at 2nd step



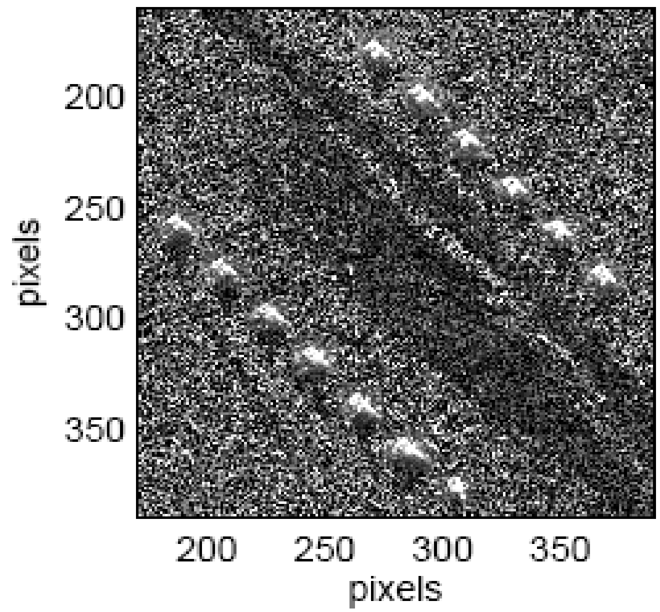
Optimal 2-step acquisition

Comparisons



(a)

Wide area SAR acquisition



(b)

Optimal two step SAR acquisition

Overall energy allocated is identical in both cases

Synergistic Activities

- Sequential resource allocation (ARO MURI)
  - Optimal dwell (temporal energy allocation) [Rangarajan07]
    - Sequential decisions framework
    - Packetization of energy with optimal stopping can provide gains of more than 5dB in MSE, 2dB in detection performance
  - Optimal search (spatial energy allocation) [Bashan07]
    - Bayesian adaptive sampling framework
    - Near-optimal energy/time allocation achievable by linear complexity algorithm
- Sparsity penalized inverse scattering (ARO MURI)
  - Bayesian likelihood model with LAZE sparsity-inducing prior
  - Optimization transfer approach combines iterative Born and sparsity inducing prior into monotone single iteration [Raich, Bagci, Michielssen]
- Multi-static SAR imaging (GD)
  - Hybrid CR bound derived for multi-static SAR [M. Davis, GD]
  - Currently being evaluated to predict performance limits: what are calibration requirements for improvement over mono-static systems

Foundations and Applications of Sensor Management

Publisher: Springer - To appear late 2007

Editors: A. Hero, D. Castanon, D. Cochran, K. Kastella

Table of contents

Doron Blatt - DRW Holdings
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  - R. Raich (Oregon State Univ)
  - S. Damelin (Georgia Southern)
  - Venkat Chandrasekeran (MIT Grad Student)

Publications (2006-2007)

- Appeared
  - R. Rangarajan, A.O. Hero and R. Raich, "Optimal sequential design of experiments for estimation in linear models," IEEE Journ. Selected Topics in Signal Processing (JSTSP), July 2007
  - R. Rangarajan, "*Resource constrained adaptive sensing* ," PhD Thesis, University of Michigan, July 2007.
- In cogitation or preparation
  - E. Bashan, R. Raich and A.O. Hero, "Efficient search under resource constraints," working draft.
  - A.O. Hero, V. Chandrasekaran, and A. Willsky, "Structured dimensionality reduction," working draft.
  - R. Raich, J. Costa, S. Damelin, and A.O. Hero, "Classification constrained dimensionality reduction," working draft.