# Energy-Efficient Multi-Robot Rendezvous: Parallel Solutions By Embodied Approximation

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## I. Introduction

Large-scale distributed systems such as animal colonies and multi-robot systems work together to solve complex tasks. We are interested in identifying mechanisms that exploit the characteristics of the embodied mulit-agent domain to solve complex computational problems. We are particularly interested in solving practical resource allocation problems in multi-robot systems, with *energy autonomy* as the key motivating problem.

The work described here exploits illustrates two mechanisms that we believe are very useful in many swarm and other large-scale multi-agent systems: (i) bearings-only control; and (ii) world-embedded computation by embodied approximation and iterated refinement.

Animals have evolved various strategies to efficiently use and obtain energy. In order to be able to work without human intervention robots should also be equipped with means to optimize the use of energy and properly replenish it.

Swarm robotics is motivated by and often uses mechanisms from real living swarms. Recent work by Loizou and Kumar studies landmark navigation based only on bearing measurements [personal communication, 2007]. The work is inspired by several species of ants which are capable of performing complex navigation without apparent use of range information. The authors show that it is possible to achieve a set of nontrivial group behaviors using only bearing measurements.

Our research considers multi-robot systems that include energy management as a first-class constraint to every aspect of their operation. Our robots must work for extended periods, and must obtain all the energy they require to work and to maintain themselves. In many applications energy is a scarce resource, so efficiency is important.

One important component of various tasks is a robot-robot rendezvous. While looking for ways to find meeting places which minimize the traveling costs for groups of robots we developed fully decentralized heuristic methods which also require no range information to converge at good approximation to the optimal solution. We believe that these heuristics afford a very simple neural implementation.

The key insight that underlies our methods is that the physical locations of the robots themselves could be considered as an approximate solution to the entire problem. An individual robot can move itself, thus refining the current solution approximation. No representation of the problem, or the current solution, needs to be held by any robot: they manifest the solution by their physical configuration. This is an example of what Payton has called "world-embedded computation" [1] and exploits the property of "strong embodiment" identified by Brooks [2], extended to a multi-agent system.

## II. Energy-efficient single-point rendezvous

Assume $n$ robots are located at positions $r_i$, $i = 1 \dots n$. When a robot moves, it expends energy proportional to the length of its trajectory. Robots have individual energy costs $c_i$ per unit of traveled distance, thus if robot $i$ moves from $a$ to $b$, it spends $c_i||a-b||$ units of energy. Now the task is to rendezvous at a point $p^*$ which minimizes the total energy spent by all robots for moving to that point:

$$p^* = \arg\min_p \sum_{i=1}^{n} c_i ||p - r_i|| \quad (1)$$

Point $p^*$ is the solution to the weighted Fermat-Torricelli problem formed by points $r_i$ with weights $c_i$ [3]. In [4] we propose and experimentally evaluate heuristics based on the following simple approach.

*Heuristic for single-point rendezvous*

Let self be the robot currently located at $r_j$. Let $d$ return a unit length vector in the direction from point $a$ to point $b$, $\vec{d}(a,b) = \frac{b-a}{||b-a||}$.

1) Calculate $\vec{D}_j = \sum_{i \neq j} c_i \vec{d}(r_j, r_i)$.
2) If $||\vec{D}_j|| < c_j$ then stop. Otherwise proceed in the direction $\vec{D}_j$.
3) Goto step 1.

These heuristics were shown to (i) produce global rendezvous; and (ii) incur travel costs only slightly greater than the global optimization method. This approach also allows to naturally incorporate new information about robot locations, and thus cope with obstacles, robot locomotion failures, etc. without invalidating previous computation. The information required by each robot is the locomotion costs and directions towards the other members of the team. No range information is needed.

## III. Energy-efficient multi-point rendezvous

Now we change the previous problem in two ways. First, there is a dedicated robot $r_0$ called a *tanker* which needs to meet with all worker robots $r_i$ to perform some service, e.g. recharge them. Second, the meetings of tanker and workers can occur at different places or the tanker can meet several workers at a single point. The only requirement is that tanker meets workers in a following preorder: it meets $r_i$ not later than it meets $r_j$ if $j > i$. Using the weighted distance locomotion cost model we want to find the meeting location which minimize the total cost incurred:

*Definition 1 (Problem formulation):* Given tanker location $p_0 \in \mathbb{R}^d$, worker locations $r_i \in \mathbb{R}^d, i = 1..k$ find

$$\min_{p_1, p_2, \ldots p_k} \sum_{i=1}^{k} w_0 ||p_{i-1} - p_i|| + w_i ||r_i - p_i|| \quad (2)$$

Here $p_i$ is a meeting point for tanker and worker $i$, $w_0$ is the cost of unit distance tanker relocation, $w_i$ gives the corresponding cost for worker $i$, $p_0$ is initial location of the tanker, $r_i$ are initial locations of the workers.

In [5] we study this problem in a more general setting and prove the following two lemmas which in some cases partially or completely solve the problem.

*Lemma 1:* If $w_i \geq 2w_0$ then $p_i^* = r_i$. If $w_i > 2w_0$, then $p_i^* = r_i$ is the unique solution for $p_i$.

*Lemma 2:* If $\sum_{i=1}^{k} w_i \leq w_0$ then $p_i^* = p_0$ for all $i$. If inequality is strict, then this solution is unique.

We also described two numerical algorithms for this problem. Both algorithms are centralized and suffer from scalability issues.

In [6] we provide a simple and fast decentralized heuristic that allows robots to meet at a good approximation to the solutions found by centralized algorithms. This heuristic is based on the following approach.

*Heuristic for multi-point rendezvous*

1) If self is worker currently located at $r_j$, then
   a) If self is the next worker to be charged($j = 1$), set $\vec{D}_1 = w_0 \vec{d}(r_1, r_0) + w_0 \vec{d}(r_1, r_2)$.
   b) Otherwise, set $\vec{D}_j = w_0 \vec{d}(r_j, r_{j-1}) + w_0 \vec{d}(r_j, r_{j+1})$.
2) If self is tanker ($j = 0$) Set $\vec{D}_0 = w_1 \vec{d}(r_0, r_1) + w_0 \vec{d}(r_0, r_2)$.
3) If $||\vec{D}_j|| < w_j$ then stop. Otherwise proceed in the direction $\vec{D}_j$.
4) Once the worker robot is met and the charged worker is not considered anymore in this instance of the problem and the workers are renumbered accordingly.
5) Go to step 1.

This approach requires no range information. The tanker needs to know the locomotion cost of itself and the first robot to be charged as well as the directions towards the first and second robots in a charging queue. Workers need to know their own locomotion costs, tanker locomotion costs and the directions towards the next and previous robot in a queue. First worker uses the direction towards tanker in place of the direction to the previous worker.

The approach is perfectly scalable since every robot per-step computation complexity is independent of the population size. The following theorem establishes correctness and bounds the running time.

*Theorem 1:* For any initial locations $r_j, j = 0, ..., k$ and meeting range $s$ if robots $r_0, r_1$ recalculate their movement direction $\vec{D}_j$ every time they travel distance $\epsilon < s/2$ then after at most $\lceil \frac{4||r_0 - r_1||^2}{\epsilon(s-2\epsilon)} \rceil$ iterations $r_0$ and $r_1$ will meet.

*Corollary 1:* For any initial locations $r_j, j = 0, ..., k$ and meeting range $s$ if robots recalculate their movement direction $\vec{D}_j$ every time they travel distance $\epsilon < s/2$ then after at most $\lceil k \frac{4C^2}{\epsilon(s-2\epsilon)} \rceil$ iterations all workers will be charged, assuming charging occurs instantaneously and $C = \max_{i,j} ||r_i - r_j||$.

We experimentally compared the paths prescribed to robots by the multi-point rendezvous heuristic and iterated application of centralized Nelder-Mead numerical optimization algorithm. Under different initial conditions heuristic manages to find qualitatively similar solutions to those of non-scalable centralized numerical optimization technique. Quantitatively heuristic performs within 10% range of a centralized numerical solution.

## IV. Summary

The scarcity of readily-available energy on Earth means that organisms have evolved to (usually) use energy with incredible efficiency. Modern evolutionary theory allows that groups of closely related animals could evolve to engage in collective energy optimization, even if that is not the cheapest strategy for any individual animal.

The heuristics we describe give examples of very simple mechanisms that can result in energy efficient group behavior. In both cases, computationally complex optimization tasks are solved using biologically affordable machinery. Thus, it seems promising to look for examples of group energy optimization in animals. Such animal strategies could and should be used to increase the energy efficiency, and eventually autonomy, of robot teams.

We suggest that bearings-only sensing and computation by embodied approximation and local refinement are useful mechanisms that can be exploited in a wide variety of natural and synthetic swarm and other multi-agent systems.

## References


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